On the Giant Component of Wireless Multihop Networks in the Presence of Shadowing

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***Abstract*—In this paper, we study transmission power to secure the connectivity of a network. Instead of requiring all nodes to be connected, we require that only a large fraction (e.g., 95%) be connected, which is called the giant component. We show that, with this slightly relaxed requirement on connectivity, significant energy savings can be achieved for a large-scale network. In particular, we assume that a total of n nodes are randomly independently uniformly distributed in a unit square in $\Re^2$, that each node has uniform transmission power, and that any two nodes are directly connected if and only if the power that was received by one node from the other node, as determined by the log-normal shadowing model, is larger than or equal to a given threshold. First, we derive an upper bound on the minimum transmission power at which the probability of having a giant component of order above qn for any fixed $q \in (0, 1)$ tends to one as $n \to \infty$. Second, we derive a lower bound on the minimum transmission power at which the probability of having a connected network tends to one as $n \to \infty$. We then show that the ratio of the aforementioned transmission power that was required for a giant component to the transmission power that was required for a connected network tends to zero as $n \to \infty$. This result implies significant energy savings if we require that only most nodes (e.g., 95%) be connected rather than requiring all nodes to be connected. This result is also applicable for any other arbitrary channel model that satisfies certain intuitively reasonable conditions.**

***Index Terms*—Connectivity, continuum percolation, giant component, log-normal shadowing model, transmission power, wireless multihop networks.**

I. Introduction

WIRELESS multihop networks, e.g., vehicular ad hoc networks, mobile ad hoc networks, and wireless sensor networks, are increasingly being used in military and civilian applications [1]. In general, a wireless multihop network consists of a group of decentralized and self-organized nodes that communicate with each other in a peer-to-peer manner over wireless channels, and packets are collaboratively forwarded hop by hop by the wireless nodes from the source to the destination with no need for base stations or any fixed infrastructure.

Connectivity is one of the most fundamental properties in wireless multihop networks [2]–[7]. A wireless multihop network is said to be connected if and only if (iff), for any pair of two nodes, there is at least one path between them. Over the past several years, the connectivity problem in wireless multihop networks has widely been investigated, and significant outcomes have been achieved [3]–[5], [7]–[11]. Nevertheless, in many real applications, it is unnecessary for all nodes to always be connected to each other [12]. Examples of such applications include a wireless sensor network for habitat monitoring [13], [14] or environmental monitoring [15], [16] and a mobile ad hoc network in which users can tolerate short off-service intervals [17], [18].

In environmental monitoring, there are scenarios where the size of the monitored phenomenon is very large (e.g., rain clouds) or the parameters (e.g., temperature, humidity) that are monitored slowly change both in space and in time. When the number of nodes for monitoring the phenomenon or measuring the parameters is very large, having a few disconnected nodes will not cause a statistically significant change in the monitored parameters. One example of such applications is a wireless sensor network that was deployed underneath the Briksdalsbreen glacier in Norway to monitor the pressure, humidity, and temperature of ice to understand glacial dynamics in response to climate change [15].

In habitat monitoring, there are scenarios where the number of objects (e.g., zebras and cane toads) that are monitored is large and where these objects are randomly almost independently distributed in the surveyed region. Having a few nodes disconnected or lost may not significantly affect the monitoring accuracy of the monitored parameter, e.g., the size or the density of the population. Examples of such applications include the experiment in [13], where a wireless acoustic sensor network was used to monitor the population distribution of invasive cane toads in northern Australia. In this application, having a few nodes disconnected has little impact on the accuracy of the estimated population distribution.

In many mobile ad hoc networks, having a number of nodes temporarily disconnected is also not critical, as long as users can tolerate short off-service intervals. For example, in

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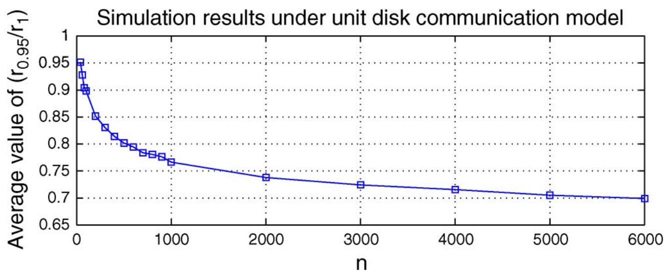

Fig. 1. Simulation results. Average value of ratio $r_{0.95}/r_1$. r_1 refers to the transmission range that was required for a connected network, and $r_{0.95}$ refers to the transmission range that was required for 95% of nodes to be connected. The ratio is the average value, and each average value is obtained over 2000 random topologies, in which a total of n nodes are uniformly randomly distributed on a unit square.

a campus-wide wireless network, students and staff can share information using wireless devices (e.g., laptops and personal digital assistants) around the campus [17]. When a wireless device temporarily loses connection, it can store the data and complete the work after becoming connected later.

Simulations show that, by allowing a small percentage of nodes to be disconnected (compared with requiring all nodes to be connected), significant energy savings can be achieved [2], [6]. As an example, Fig. 1 shows simulation results that compare the transmission range that was required for all nodes to be connected with the transmission range that was required for 95% of nodes to be connected in a network of n randomly independently uniformly distributed nodes in a unit square based on a simple channel model, i.e., the *unit disk communication model*, where two nodes can directly communicate with each other iff their Euclidean distance is below a given threshold, which is usually referred to as the *transmission range*[1] [2], [3]. Then, comparable simulation results under the more-realistic channel model in this paper will be provided in Section IV-A. As shown in Fig. 1, when the number of nodes is 1000, the transmission range that was required for 95% nodes to be connected is 24% less than the transmission range that was required for a connected network. Based on a conservative estimate that the required transmission power increases with the square of the required transmission range, this result yields energy savings of at least 42%. In addition, the ratio decreases as the total number of nodes n increases. As we will show in Section IV, the ratio will go to zero when $n \to \infty$. This result means that the energy savings are even more significant in a network with a larger number of nodes. Many real applications do not require all nodes to be connected; thus, it is appropriate to consider slightly relaxing the connectivity requirement, i.e., requiring most nodes (e.g., 95%) to be connected rather than requiring all nodes to be connected, to achieve significant savings in power consumption. It then becomes important to investigate the largest connected component that contains a nonvanishing fraction of nodes, which is termed the *giant component* [6], [19], [20]. A formal definition of the giant component will be given in Section III.

[1]As we will later introduce in Section III, this channel model is a special case of the more-realistic channel model in this paper.

In this paper, we analytically investigate the giant component by employing the *log-normal shadowing model* [21] to see how a weaker requirement on network connectivity can achieve considerable reductions in the transmission power (i.e., energy cost). In particular, we assume that a total of n nodes are randomly independently uniformly distributed in a unit square in $\Re^2$ and that all nodes have the same transmission power. Any two nodes are directly connected iff the power that was received by one node from the other node, as determined by the log-normal shadowing model, is larger than or equal to a given threshold. In this paper, we have ignored the impact of other more complicated factors (e.g., interference and network traffic distribution) to focus on the main theme of this paper. In addition, we consider only the energy that was consumed on radio frequency transmissions [2], [6], [19], [22]. The node-placement assumption is widely used by many researchers [2]–[4], [7], [9], [12], [23]. The log-normal shadowing model is chosen, because it can better capture the shadowing effects and is more realistic than the unit disk communication model, which has widely been used in the literature [6], [24]–[26]. The goal is to find an analytical *upper bound* on the minimum transmission power that was required to have a giant component of order above qn for any fixed $q \in (0, 1)$ (see Theorem 1) and an analytical *lower bound* on the minimum transmission power that was required to have a connected network (see Theorem 2); for both bounds, n must be large. Based on these two results, we show that the minimum transmission power that was required to have a giant component is vanishingly small compared with the minimum transmission power that was required to have a connected network as $n \to \infty$ (Corollary 1). This result means that significant energy savings can be achieved if we only require most nodes (e.g., 95%) to be connected rather than requiring all nodes to be connected, particularly in a network with a large number of nodes. In addition, as a helpful by-product, the interference can also be reduced by using a reduced transmission power [27]. The results (e.g., Corollary 1) of this paper, which were obtained under the log-normal shadowing model, are also applicable for other channel models that satisfy certain intuitively reasonable conditions. Details of this model will be given in Section VII. To the best of our knowledge, our results have not previously been reported.

The rest of this paper is organized as follows. Section II briefly reviews related work. Section III describes the network model and some basic concepts of graph theory that were used in this paper. In Section IV, we present the main results (Theorems 1 and 2 and Corollary 1) of this paper. In Section V, we prove Theorem 1. In Section VI, we prove Theorem 2. In Section VII, we consider extensions of the results for other channel models. Finally, Section VIII concludes this paper and discusses future research directions.

II. Related Work

The concept of the giant component has extensively been investigated in the literature for *Bernoulli random graphs* [28], and an analytical formula that relates the giant component size and the average node degree has been found [28], [29]. The giant component size is defined as the ratio of the number of nodes in the giant component to the total number of nodes, and the average node degree is the average number of neighbors of an arbitrary node. However, it is well known that the Bernoulli random graph is not suitable for modeling wireless multihop networks; hence, it is inappropriate to directly apply the results on the giant component from Bernoulli random graphs into wireless multihop networks.

In contrast to the Bernoulli random graph, some more suitable models (e.g., *geographical threshold graphs* [30] and *geometric random graphs* [2], [19]) were introduced and used to study the giant component in wireless multihop networks.

In [30], Bradonjiæ *et al.* studied the giant component based on the geographical threshold graph, where n nodes are randomly uniformly distributed in a bounded area, and the existence of a link between any two nodes is determined by both the Euclidean distance between them and the node weights that were assigned to them. The authors derived conditions for the absence and existence of a giant component. Németh *et al.* [31] *empirically* investigated the giant component size by using a fractal propagation model where the probability of having a link between two nodes is determined by their Euclidean distance and two nonnegative constants. They found that the giant component size can be characterized by a single parameter, i.e., the average node degree.

Using the geometric random graph,[2] Raghavan *et al.* [19] proposed an empirical formula for the minimum transmission range at which a 2-D wireless sensor network has a giant component with a high probability and showed through simulations that the minimum transmission range is approximately inversely proportional to $\sqrt{n}$. Using the same network model, Santi *et al.* [2] *empirically* investigated the minimum transmission range that ensures either a connected network or a giant component that contains a large fraction (e.g., 90%) of nodes with a high probability. The authors showed through simulations that considerable reductions of the transmission range (i.e., of the energy cost) can be achieved if we only require a large percentage of nodes to be connected to a single component. A similar conclusion can be found in [32]. In [22], Rahnavard *et al.* applied the same network model and proposed an energy-efficient two-phase broadcast scheme using known results (i.e., critical node density for the occurrence of a giant component) for the giant component in wireless sensor networks. In the first phase, this scheme makes sure that a giant component receives data packets, and in the second phase, this scheme makes sure that all nodes receive the data. The authors showed that this approach is more energy efficient than requiring all nodes to receive the data in only one phase.

In [6], Hekmat *et al.* employed a more-realistic channel model, i.e., the log-normal shadowing model, to *empirically* investigate the giant component size. The authors assumed that a total of n nodes are randomly uniformly distributed in a square and that a link exists between two nodes if the power that was received at one node from the other node, as determined by the log-normal shadowing model, is greater than a given threshold. Based on the analytical results from Bernoulli random graphs, the authors proposed an empirical formula that relates the giant component size and the average node degree. The authors also showed through simulations that significant energy savings can be achieved by requiring that only a large percentage of nodes are connected.

The results with regard to the giant component in [2], [6], [19], and [32] are all obtained based on simulation studies. In addition, [2], [19], and [32] study the giant component by employing the aforementioned unit disk communication model. The unit disk communication model is based on only the path-loss phenomenon [21] and assumes that the received signal strength at a receiving node from a transmitting node is only determined by a deterministic function of the Euclidean distance between the two nodes. However, in reality, the received signal strength often shows probabilistic variations that were induced by shadowing effects that are unavoidably caused by different levels of clutter on the propagation path [21], [24]. To better capture physical reality, one should consider the variations of the received signal strength. It has been shown in [33] and [34] that a more-accurate modeling of the physical layer is important for better understanding of wireless multihop network characteristics. These observations motivate us to analytically investigate the giant component by employing a more-realistic channel model.

In this paper, we shall prove that the minimum transmission power that was required to have a giant component of order above qn $(0 < q < 1)$ is vanishingly small compared with the minimum transmission power that was required to have a connected network as $n \to \infty$ by using the log-normal shadowing model, which is more realistic than the unit disk communication model in [2] and [19].

III. Preliminaries

A. Wireless-Channel Model

The wireless received signal strength $P_r(d_{uv})$ between any two nodes u and v has popularly been modeled by a log-normal shadowing model [21], [24], [35], i.e.,

$$P_r(d_{uv}) = P_t - PL_0(d_0) - 10\alpha \log_{10} \frac{d_{uv}}{d_0} + Z_\sigma \qquad (1)$$

where $P_r(d_{uv})$ is the received power at a receiving node v from a transmitting node u (in decibel milliwatts), P_t is the transmitted power of the transmitting node u (in decibel milliwatts), d_{uv} is the Euclidean distance between nodes u and v, $PL_0(d_0)$ is the reference path loss (in decibels) at a reference distance d_0, α is the path-loss exponent that indicates the rate at which the received signal strength decreases with distance, and Z_σ is a zero-mean Gaussian (normal) random variable (in decibels) with standard deviation σ (also in decibels). The reference path loss $PL_0(d_0)$ is calculated using the free-space Friis equation or is obtained through field measurements at distance d_0 [21]. In this paper, $PL_0(d_0)$ and d_0 are assumed to be known constants [26], [36]. The value of α depends on the environment and terrain structure and can vary between 2 in free space and 6 in heavily built urban areas. The value of σ is usually larger than zero and can be as high as 12 decibels [21].

[2]A geometric random graph is typically formed by randomly uniformly distributing n nodes in a bounded area (e.g., a unit square) and connecting any two nodes iff their Euclidean distance is below a given threshold.

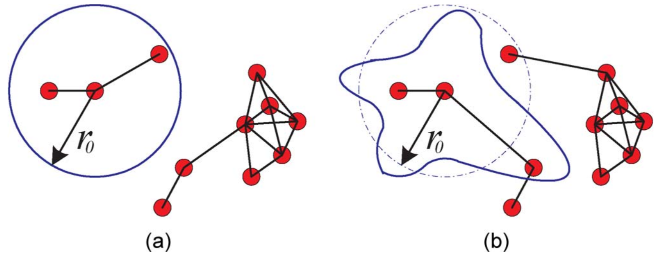

Fig. 2. Two channel models and resulting network topologies. r_0 is the transmission range in the absence of shadowing. (a) Unit disk communication model. (b) Log-normal shadowing model.

In addition to the log-normal shadowing model, we will make the following three assumptions in this paper, which are commonly used by many researchers [6], [24], [25], [35], [37].

1) α remains constant in the network area.
2) The shadow-fading attenuations between all pairs of nodes are independent identically distributed (i.i.d.), and σ remains constant for all pairs of nodes.[3]
3) The paths are symmetric, i.e., the received power at node v from node u is equal to the received power at node u from node v.

For any two nodes u and v, there exists a wireless link between them (or they are directly connected) iff the received power $P_r(d_{uv})$ is not less than some given threshold P_{th} (also in decibel milliwatts), i.e., $P_r(d_{uv}) \geq P_{th}$ [6], [21], [24]. Hence, using (1), the probability that two random nodes u and v that were separated by a known distance x are directly connected, which is denoted as $\mathbb{P}(x)$, is given by

$$\mathbb{P}(x) = \Pr\{P_r(x) \geq P_{th}\} = \int_{10\alpha \log_{10} \frac{x}{r_0}}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}} dz \quad (2)$$

where

$$r_0 = d_0 \times 10^{\frac{P_t - PL_0(d_0) - P_{th}}{10\alpha}} \quad (3)$$

is the transmission range in the absence of shadowing (i.e., $\sigma = 0$). In this paper, we consider that all nodes have the same transmission power P_t. It is clear that the shadowing-free transmission range r_0 is related to the transmission power P_t by (3). Throughout this paper, we shall investigate the transmission power P_t by investigating r_0, given that P_{th} and α are fixed.

When $\sigma = 0$, there is no shadowing; the received power $P_r(d_{uv})$ is then a deterministic function of the Euclidean distance d_{uv} between u and v. The channel model reduces to the unit disk communication model, where each node has a circular transmission area. When $\sigma > 0$, the received power $P_r(d_{uv})$ is determined by both the deterministic function of the Euclidean distance d_{uv} between u and v and the shadowing. The transmission area of each node is no longer a circular area. Fig. 2 illustrates the two channel models and the corresponding network topologies. In this paper, our focus is on the more-realistic log-normal shadowing model, because in real applications, σ is larger than zero.

[3]Although field measurements in real applications seem to indicate that the shadow fades between two links with a common node are correlated [35], this i.i.d. assumption is generally considered appropriate for far-field transmissions and is widely used in the literature [6], [24], [25], [35], [37].

B. Network Model

In general, a wireless multihop network can be represented by an undirected graph $G = (V, E)$ with a set of vertices $V = V(G)$ and a set of edges $E = E(G)$. Each vertex of the set V uniquely represents a node, each edge of the set E uniquely represents a wireless link, and *vice versa*. The graph $G = (V, E)$ is then called the *underlying graph* of the network. In the following discussion, we give a formal definition of the underlying graph of the network. Denote this underlying graph as $G(\mathcal{X}_n, r_0, \sigma)$.

Definition 1: Let $X_1, X_2, \ldots, X_n$ be n points that are randomly independently uniformly distributed in a unit square in $\Re^2$, and let $\mathcal{X}_n = \{X_1, X_2, \ldots, X_n\}$. The underlying graph $G(\mathcal{X}_n, r_0, \sigma)$ is an undirected graph with $\mathcal{X}_n$ as its vertex set and an edge that connects each pair of vertices X_i and X_j in $\mathcal{X}_n$ with probability $\mathbb{P}(\|X_i - X_j\|)$, where r_0 is given by (3), σ is the standard deviation of the shadowing in the log-normal shadowing model, function $\mathbb{P}(\cdot)$ is given by (2), and norm $\|\cdot\|$ refers to the Euclidean norm.

Remark: Although the network model in this paper is built on a unit square, all results that were developed for a square of unit size can easily be extended to a square of arbitrary size. In fact, by a suitable space rescaling [20], [38], all the properties in this paper can be reformulated [9], [20], [39].

C. Notation

This paragraph recalls some basic concepts from graph theory [38], [40]. Two nodes are neighbors (or are directly connected) iff they have a wireless link between each other. The *degree* of a node u, which is denoted as $d(u)$, is the number of its neighbors. A node of degree zero is called an *isolated node*. A graph is connected iff, for any pair of vertices, there is at least one path between them. A *component* of a graph is a maximally connected subgraph of the graph. The *order* of a component is the total number of vertices in the component. The largest component that contains a nonvanishing fraction of vertices is called the *giant component*. In the following discussion, the order of the giant component in a graph G is denoted by $L(G)$.

Throughout this paper, we will use standard mathematical notations [41] with regard to the following asymptotic behavior of functions.

1) $y(n) = o(g(n))$ if $\lim_{n\to\infty}(y(n)/g(n)) = 0$.
2) $y(n) \ll g(n)$ or $g(n) \gg y(n)$ if $y(n) = o(g(n))$.
3) $y(n) \sim g(n)$ if $\lim_{n\to\infty}(y(n)/g(n)) = 1$.
4) An event ξ_n (depending on the value of n) is said to *asymptotically almost surely* (a.a.s.) occur if its probability tends to one as $n \to \infty$.

Throughout this paper, let η be a constant given by $\eta = (\log 10/10)$. Define an n-dependent integer set J_n as

$$J_n := \left\{ j : j \in \mathbb{N},\ \left\lfloor n - 2n^{\frac{3}{4}} \right\rfloor \leq j \leq n \right\} \quad (4)$$

where $\mathbb{N}$ represents the set of positive integers.

IV. Main Results

In this section, we present the main results, i.e., an asymptotic analytical *upper bound* on the minimum transmission power for the giant component (see Theorem 1), an asymptotic analytical *lower bound* on the minimum transmission power for connectivity (see Theorem 2), and a comparison between the minimum transmission power for giant component and the minimum transmission power for connectivity (see Corollary 1). The proofs of Theorems 1 and 2 are deferred to the next two sections, respectively. To avoid complexity in the derivation, we will ignore the boundary effect that was caused by nodes that are close to the boundary of the network area. In Section IV-A, we will provide some simulation results to evaluate the impact of the boundary effect.

Our main result for the upper bound on the minimum transmission power for the giant component is given in the following theorem.

Theorem 1: Consider $G(\mathcal{X}_n, r_0, \sigma)$ in $\Re^2$. Let q be any fixed real number within (0, 1). Let c be any fixed real number. Let $f(n)$ be a function of n that satisfies

$$f(n) > 0, \quad \lim_{n\to\infty} f(n) = \infty, \quad \lim_{n\to\infty} \frac{f(n)}{\log n} = 0. \tag{5}$$

Ignore the boundary effect. If $\pi r_0^2 \exp((2\eta^2\sigma^2)/(\alpha^2)) = ((f(n) + c)/n)$, then

$$\lim_{n\to\infty} \Pr\left\{L\left(G(\mathcal{X}_n, r_0, \sigma)\right) \geq qn\right\} = 1.$$

Theorem 1 says that, if r_0 satisfies $\pi r_0^2 \exp((2\eta^2\sigma^2)/(\alpha^2)) = ((f(n) + c)/n)$, the network will a.a.s. have a giant component of order above qn as $n \to \infty$. It provides an upper bound on the minimum transmission power that was required to have a giant component of order above qn. In [42], we have derived similar result with regard to the upper bound in the nonshadowing case (i.e., $\sigma = 0$), which is $\pi r_0^2 = (f(n) + c/n)$. Hence, the difference between the shadowing and the nonshadowing cases is that there is no exponential term, i.e., $\exp((2\eta^2\sigma^2)/(\alpha^2))$, in the nonshadowing case. Notice that having a log-normal shadowing model rather than a unit disk communication model allows a reduction in the value of r_0 for a fixed large n, i.e., the random variation associated with the log-normal shadowing model is helpful.

Remark: At first glance, the result in Theorem 1 appears abnormal, because it suggests the probability of having a giant component of order qn, because $n \to \infty$ is independent of q. Here, we offer the following intuitive explanation for the result. It is well known that the width of the phase-transition region from an almost-disconnected network to an almost-connected network approaches zero as $n \to \infty$ [43]. This case means that, at large n, the probability of having a connected network as a function of the transmission power is almost like a step function such that, at a certain value of the transmission power (termed the critical transmission power), a tiny variation in the transmission power causes a large change in the probability. The aforementioned result indicates that the same phenomenon may also be observed for the probability of having a giant component. Possibly, a refined set of conditions of $f(n)$ in (5) can allow for distinguishing the different values of q.

Our main result for the lower bound on the minimum transmission power for connectivity is given in the following theorem.

Theorem 2: Let $\mathbb{P}_d(\mathcal{X}_n, r_0, \sigma)$ denote the probability that the graph $G(\mathcal{X}_n, r_0, \sigma)$ in $\Re^2$ is disconnected. Let c be any fixed real number. Ignore the boundary effect. If $\pi r_0^2 \exp((2\eta^2\sigma^2)/(\alpha^2)) = ((\log n + c)/n)$, then

$$\liminf_{n\to\infty} \mathbb{P}_d(\mathcal{X}_n, r_0, \sigma) \geq 1 - \exp(-e^{-c}).$$

Observe that $c < \infty$, i.e., the lower bound given in the aforementioned inequality is always positive, which implies a nonzero probability of having a disconnected network as $n \to \infty$. Hence, to a.a.s. have a connected network, r_0 must at least satisfy the condition in Theorem 2, i.e., $\pi r_0^2 \exp((2\eta^2\sigma^2)/(\alpha^2)) = ((\log n + c)/n)$. Indeed, Theorem 2 provides a lower bound on the minimum transmission power that was required to have a connected network. Theorem 2 turns out to have a similar form to the widely cited result in [3] for the nonshadowing case, i.e., $\pi r_0^2 = ((\log n + c)/n)$. Note that Theorem 2 has indirectly been derived in [24], [37], and [44] for nodes that were distributed according to a homogeneous Poisson point process.

Based on Theorems 1 and 2, we can obtain the following important result.

Corollary 1: Let q be any fixed real number within (0, 1). Let R_q be the critical value of r_0 that was required to a.a.s. have a giant component of order above qn and let R_1 be the critical value of r_0 required to a.a.s. have a connected network. Ignore the boundary effect. Then

$$\lim_{n\to\infty} \frac{R_q}{R_1} = 0.$$

Proof: Theorem 1 provides an upper bound on R_q, and Theorem 2 provides a lower bound on R_1. Hence, we have

$$\lim_{n\to\infty} \frac{R_q}{R_1} \leq \lim_{n\to\infty} \frac{\sqrt{\frac{f(n)+c}{n\pi \exp\left(\frac{2\eta^2\sigma^2}{\alpha^2}\right)}}}{\sqrt{\frac{\log n + c'}{n\pi \exp\left(\frac{2\eta^2\sigma^2}{\alpha^2}\right)}}} = \lim_{n\to\infty} \sqrt{\frac{f(n)+c}{\log n + c'}} = 0$$

where c and c' are any fixed real numbers. ■

The implication of the aforementioned result is that, when $n \to \infty$, the transmission power that was required to have a giant component is vanishingly small compared with the transmission power that was required to have a connected network. Therefore, in a large-scale network, significant energy savings can be achieved by requiring most nodes, instead of all nodes, to be connected. Furthermore, in a network where almost (but not) all nodes are connected, a large leap in transmission power may be required to connect the remaining few nodes, and the transmission power that was required for a large-scale network to be connected is dominated by these few nodes, i.e., rare events. In many real applications, it is not worthwhile to substantially increase the transmission power to connect the remaining few nodes [2], and by only requiring a giant component, we can achieve significant energy savings and a much longer lifetime of the network.

Remark: As mentioned in Section III-B, the results, i.e., Theorems 1 and 2 and Corollary 1, can easily be reformulated by using the space-rescaling technique [20, Th. 9.17]. In particular, Corollary 1 will still hold for a network that was deployed on a square of arbitrary size without any change in formation. According to [20, Th. 9.17], a network with n randomly uniformly distributed nodes on a square of size $\sqrt{A} \times \sqrt{A}$ and with a shadowing-free transmission range $\mathbb{R}_0 = \sqrt{A}r_0$ has the same statistical connectivity property as a network with n randomly uniformly distributed nodes on a unit square and with a shadowing-free transmission range r_0, i.e., the network in this paper. The requirements on r_0 can therefore be translated into the requirements on $\mathbb{R}_0$ by the formula $\mathbb{R}_0 = \sqrt{A}r_0$. In particular, in the network that was distributed on a square of size $\sqrt{A} \times \sqrt{A}$, the ratio between $\mathbb{R}_q$ and $\mathbb{R}_1$ (with the same meaning as R_q and R_1, respectively) is given by

$$\frac{\mathbb{R}_q}{\mathbb{R}_1} = \frac{\sqrt{A}R_q}{\sqrt{A}R_1} = \frac{R_q}{R_1} \to 0, \qquad \text{as } n \to \infty.$$

A. Simulation Study

In this paper, we have ignored the boundary effect to avoid complexity in the derivation. To evaluate the impact of the boundary effect, we conducted a simulation study, considering the boundary effect, to check the validity of Corollary 1, which is the central contribution of this paper. In the following discussion, we report the simulation results (with the boundary effect), which compare $r_{0.95}$ with r_1 under the log-normal shadowing model, where $r_{0.95}$ is the minimum value of r_0 that was required for 95% of nodes to be connected, and r_1 is the minimum value of r_0 that was required for all nodes to be connected. We also compare these simulation results with the result in Section I (see Fig. 1) under the unit disk communication model.

Fig. 3(a) shows the average value of the ratio between $r_{0.95}$ and r_1 when $\alpha = 2$ and $\sigma = 0,\ 1,$ and 3, where $\sigma = 0$ represents the unit disk communication model. As shown in Fig. 3(a), the ratio is always smaller than one, and for fixed α and σ, the ratio decreases as the total number of nodes n increases in the presence of the boundary effect. There are several further observations in Fig. 3(a). As explained in Section I, if the path-loss exponent α increases, the energy savings will be much greater. We can see that, for fixed α and n, the ratio for a lower value of σ is larger than that for a higher value of σ, which means that more reduction in transmission power (i.e., more energy savings) can be achieved for a higher value of σ. The figure also indicates that, for fixed α and n, the variation of the ratio with σ is not linear. This result should be expected. As Theorems 1 and 2 show, for fixed n, r_0 does not linearly depend on σ but in proportion to $\exp((2\eta^2\sigma^2)/(\alpha^2))$.

To further investigate the validity of Corollary 1, we also present simulation results (with the boundary effect) obtained with different shapes of the network area (e.g., rectangle and circle). Fig. 3(b) shows the average value of the ratio between $r_{0.95}$ and r_1 when $\alpha = 2$ and $\sigma = 3$ for three different shapes of network area: 1) unit square; 2) unit-area rectangle $2 \times (1/2)$; and 3) unit-area disc. We can see that the difference between them is marginal and can be ignored for $n = 40 \sim 6000$.

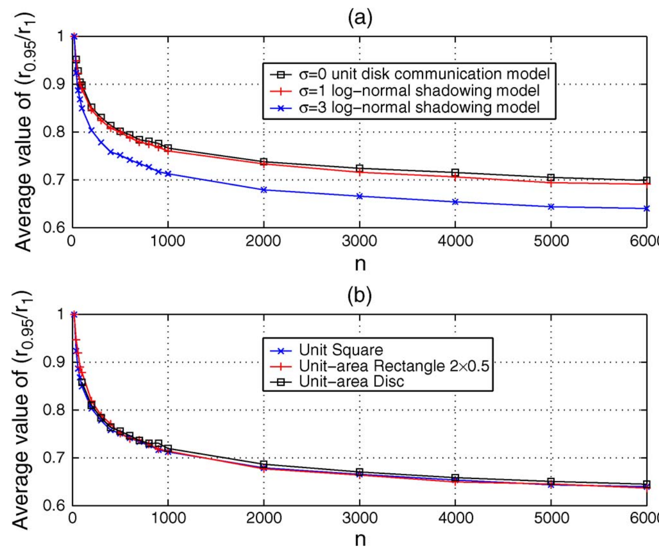

Fig. 3. Simulation results with the Boundary effect. Average value of ratio $r_{0.95}/r_1$. $r_{0.95}$ refers to the minimum value of r_0 that was required for 95% of nodes to be connected. r_1 refers to the minimum value of r_0 that was required for all nodes to be connected. The ratio is the average value, and each average value is obtained over 2000 random topologies, in which a total of n nodes are uniformly randomly distributed on a unit square, unit-area rectangle $2 \times (1/2)$, or unit-area disc. (a) With boundary effects; unit square. $\alpha = 2$. (b) With boundary effects. $\sigma = 3$, and $\alpha = 2$.

Based on Fig. 3(a) and (b) and the aforementioned discussion, we conjecture that Corollary 1 is also valid when taking the boundary effect into account. In other words, the boundary effect may only affect the convergence rate of Corollary 1 but may not affect the conclusion of Corollary 1, i.e., $(R_q/R_1) \to 0$ as $n \to \infty$.

V. Proof of Theorem 1

In this section, we shall prove Theorem 1, which provides an asymptotic analytical upper bound on the minimum transmission power at which the probability of having a giant component of order above qn tends to one as $n \to \infty$, where q is any fixed real number in (0, 1).

To derive the results, we will use Poissonization and de-Poissonization techniques, considering that "Poissonization is a key technique in geometric probability" [20, p. 18]. Let $\{X_1, X_2, X_3, \ldots\}$ be an infinite series of points that are randomly independently uniformly distributed in a unit square in $\Re^2$. Given $\lambda > 0$, let N_λ be a Poisson random variable with mean λ, independent of $\{X_1, X_2, X_3, \ldots\}$, and let

$$\mathcal{P}_\lambda := \{X_1, X_2, \ldots, X_{N_\lambda}\}. \tag{6}$$

Then, $\mathcal{P}_\lambda$ is the restriction to a unit square of a Poisson point process with intensity λ in $\Re^2$ [20]. The point process $\mathcal{P}_\lambda$ has a spatial independence property; thus, it is easier to work with the graph $G(\mathcal{P}_\lambda, r_0, \sigma)$ rather than with $G(\mathcal{X}_n, r_0, \sigma)$ [20], where the graph $G(\mathcal{P}_\lambda, r_0, \sigma)$ is obtained in the same way as in $G(\mathcal{X}_n, r_0, \sigma)$, except that the vertex set is $\mathcal{P}_\lambda$ instead of $\mathcal{X}_n$. With $\mathcal{P}_\lambda$ and $\mathcal{X}_n$ being related, we shall start by proving results about $\mathcal{P}_\lambda$ and then deduce results about $\mathcal{X}_n$ from these findings. The first instance of this result occurs in Lemma 6.

We will frequently use the following lemmas in later derivations.

Lemma 1 [3]: For any $x \in [0, 1]$, we have

$$(1-x) \leq e^{-x}.$$

Lemma 2: Suppose that M_n is a Poisson random variable with an expected value $E(M_n) = \lfloor n - n^{3/4} \rfloor$. Then

$$\lim_{n\to\infty} \Pr\left\{ \left\lfloor n - 2n^{\frac{3}{4}} \right\rfloor \leq M_n \leq n \right\} = 1.$$

Proof: M_n is a Poisson random variable with mean $E(M_n) = \lfloor n - n^{3/4} \rfloor$; thus, its variance, denoted as $D^2(M_n)$, is also $\lfloor n - n^{3/4} \rfloor$. By Chebyshev's inequality, for any $\varepsilon > 0$, we have

$$\Pr\left\{ |M_n - E(M_n)| \geq \varepsilon \right\} \leq \frac{D^2(M_n)}{\varepsilon^2}.$$

Based on the aforementioned equation, we can obtain that

$$\Pr\left\{ E(M_n) - \varepsilon \leq M_n \leq E(M_n) + \varepsilon \right\} \geq 1 - \frac{D^2(M_n)}{\varepsilon^2}. \tag{7}$$

Now, let $\varepsilon = \lfloor n^{3/4} \rfloor$. Substituting the value of $E(M_n)$, $D^2(M_n)$ and ε into (7), we have

$$\Pr\left\{ \left\lfloor n - n^{\frac{3}{4}} \right\rfloor - \left\lfloor n^{\frac{3}{4}} \right\rfloor \leq M_n \leq \left\lfloor n - n^{\frac{3}{4}} \right\rfloor + \left\lfloor n^{\frac{3}{4}} \right\rfloor \right\}$$
$$\geq 1 - \frac{\left\lfloor n - n^{\frac{3}{4}} \right\rfloor}{\left\lfloor n^{\frac{3}{4}} \right\rfloor^2} \sim 1 - o(1), \qquad \text{as } n \to \infty. \tag{8}$$

For any two positive real numbers a and b, it is clear that

$$\lfloor a - b \rfloor \leq \lfloor a \rfloor - \lfloor b \rfloor, \quad \lfloor a \rfloor + \lfloor b \rfloor \leq a + b.$$

Hence, we have

$$\left\lfloor n - 2n^{\frac{3}{4}} \right\rfloor = \left\lfloor \left(n - n^{\frac{3}{4}}\right) - n^{\frac{3}{4}} \right\rfloor \leq \left\lfloor n - n^{\frac{3}{4}} \right\rfloor - \left\lfloor n^{\frac{3}{4}} \right\rfloor$$
$$\left\lfloor n - n^{\frac{3}{4}} \right\rfloor + \left\lfloor n^{\frac{3}{4}} \right\rfloor \leq n - n^{\frac{3}{4}} + n^{\frac{3}{4}} = n.$$

Thus, we have

$$\Pr\left\{ \left\lfloor n - n^{\frac{3}{4}} \right\rfloor - \left\lfloor n^{\frac{3}{4}} \right\rfloor \leq M_n \leq \left\lfloor n - n^{\frac{3}{4}} \right\rfloor + \left\lfloor n^{\frac{3}{4}} \right\rfloor \right\}$$
$$\leq \Pr\left\{ \left\lfloor n - 2n^{\frac{3}{4}} \right\rfloor \leq M_n \leq n \right\}. \tag{9}$$

By (8) and (9), the result follows. ■

To prove Theorem 1, Lemmas 3–6 are needed. Lemma 6 is used to prove Theorem 1, Lemma 3 is used to prove Lemma 4, and Lemmas 4 and 5 are used to prove Lemma 6.

Lemma 3: Let $\mathbb{P}(\mathcal{X}_n, r_0, \sigma)$ be the probability that two randomly selected nodes in $G(\mathcal{X}_n, r_0, \sigma)$ in $\Re^2$ are directly connected. Assume that $r_0 \ll 1$. Then

$$\mathbb{P}(\mathcal{X}_n, r_0, \sigma) \sim \pi r_0^2 \exp\left(\frac{2\eta^2\sigma^2}{\alpha^2}\right)$$

where $\eta = (\log 10/10)$.

Proof: Let X be the random variable that represents the Euclidean distance between any two randomly selected nodes in $G(\mathcal{X}_n, r_0, \sigma)$. Nodes are uniformly independently distributed in a unit square; thus, the probability density function of X is given by [45], [46]

$$p_X(x) = \begin{cases} 2\pi x - 8x^2 + 2x^3, & 0 \leq x \leq 1 \\ 2\sqrt{x^2-1} - \frac{x^2+2}{2} \\ + \sin^{-1}\left(\frac{1}{x}\right) - \cos^{-1}\left(\frac{1}{x}\right), & 1 < x \leq \sqrt{2}. \end{cases}$$

Hence, based on (1) and (3), we have,

$$\begin{aligned}
&\mathbb{P}(\mathcal{X}_n, r_0, \sigma) \\
&= \Pr\left\{ Z_\sigma \geq 10\alpha \log_{10} \frac{X}{r_0} \right\} \\
&= \int_{-\infty}^{\infty} \Pr\left\{ X \leq r_0 \exp\left(\frac{\eta z}{\alpha}\right) \right\} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}} dz \\
&= \int_{-\infty}^{\infty} \left[\int_0^{\min\left\{1, r_0 \exp\left(\frac{\eta z}{\alpha}\right)\right\}} p_X(x) dx \right] \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}} dz \\
&\quad + \int_{-\infty}^{\infty} \left[\int_{\min\left\{1, r_0 \exp\left(\frac{\eta z}{\alpha}\right)\right\}}^{\min\left\{\sqrt{2}, r_0 \exp\left(\frac{\eta z}{\alpha}\right)\right\}} p_X(x) dx \right] \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}} dz.
\end{aligned}$$

Because $r_0 \ll 1$, we have $\min\{1, r_0 \exp(\eta z/\alpha)\} = r_0 \exp(\eta z/\alpha)$ and $\min\{\sqrt{2}, r_0 \exp(\eta z/\alpha)\} = r_0(\eta z/\alpha)$ for all sufficiently small r_0. Therefore, we have

$$\begin{aligned}
&\mathbb{P}(\mathcal{X}_n, r_0, \sigma) \\
&\sim \int_{-\infty}^{\infty} \left[\int_0^{r_0 e^{\frac{\eta z}{\alpha}}} (2\pi x - 8x^2 + 2x^3) dx \right] \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}} dz \\
&= \pi r_0^2 \exp\left(\frac{2\eta^2\sigma^2}{\alpha^2}\right) - \frac{8}{3} r_0^3 \exp\left(\frac{9\eta^2\sigma^2}{2\alpha^2}\right) \\
&\quad + \frac{1}{2} r_0^4 \exp\left(\frac{8\eta^2\sigma^2}{\alpha^2}\right) \\
&= \pi r_0^2 \exp\left(\frac{2\eta^2\sigma^2}{\alpha^2}\right) (1 + o(1)).
\end{aligned}$$

The result follows. ■

Lemma 4: Let j be any integer that satisfies $\lfloor n - 2n^{3/4} \rfloor \leq j \leq n$. Let $I(j, r_0, \sigma)$ be the number of isolated vertices in graph $G(\mathcal{X}_j, r_0, \sigma)$ in $\Re^2$. Let q be any fixed real number within (0, 1), and let c be any fixed real number. Let $f(n)$ be a function of n that satisfies (5). Ignore the boundary effect. If $\pi r_0^2 \exp((2\eta^2\sigma^2)/(\alpha^2)) = ((f(n)+c)/n)$, then

$$\lim_{n\to\infty} \Pr\{I(j, r_0, \sigma) \geq j - qn + 1\} = 0.$$

Proof: In this paper, we assume that links between nodes are independent of each other [3], [24], [37] (see earlier assumptions in Section III-A). Therefore, ignoring the boundary effect and using Lemma 3, the probability that an arbitrary node in $G(\mathcal{X}_j, r_0, \sigma)$ is isolated, which is denoted by $\mathbb{P}_{iso}(j)$, is given by

$$\begin{aligned}\mathbb{P}_{iso}(j) &\sim [1 - \mathbb{P}(\mathcal{X}_j, r_0, \sigma)]^{j-1} \\ &\sim \left[1 - \pi r_0^2 \exp\left(\frac{2\eta^2\sigma^2}{\alpha^2}\right)\right]^{j-1}.\end{aligned}$$

Let $E(I(j, r_0, \sigma))$ denote the expected value of $I(j, r_0, \sigma)$. By the Palm theory [20, Th. 1.6], we have

$$\begin{aligned}E\left(I(j, r_0, \sigma)\right) &= j \times \mathbb{P}_{iso}(j) \\ &= j \times \left[1 - \pi r_0^2 \exp\left(\frac{2\eta^2\sigma^2}{\alpha^2}\right)\right]^{j-1}.\end{aligned}$$

By Lemma 1, we have

$$E\left(I(j, r_0, \sigma)\right) \leq j \times e^{-(j-1)\pi r_0^2 \exp\left(\frac{2\eta^2\sigma^2}{\alpha^2}\right)} = j \times \left(\frac{e^{-c}}{e^{f(n)}}\right)^{\frac{j-1}{n}}.$$

Because $\lfloor n - 2n^{3/4} \rfloor \leq j \leq n$, we have $(j - 1/n) \to 1$ and $(j/j - qn + 1) \to (1/1 - q)$ as $n \to \infty$ and $(j - qn + 1) > 0$ for all sufficiently large n. Hence, by the Markov inequality, it follows that

$$\begin{aligned}\Pr\{I(j, r_0, \sigma) \geq j - qn + 1\} &\leq \frac{E\left(I(j, r_0, \sigma)\right)}{j - qn + 1} \\ &\leq \frac{j}{j - qn + 1}\left(\frac{e^{-c}}{e^{f(n)}}\right)^{\frac{j-1}{n}} \\ &= o(1), \quad \text{as} \quad n \to \infty.\end{aligned}$$

Therefore, the result immediately follows. ■

The previous lemmas applied to graphs that were associated with uniform distribution of nodes. Now, we obtain two results that apply when there is Poisson distribution.

Lemma 5: Consider $G(\mathcal{P}_{m(n)}, r_0, \sigma)$ in $\Re^2$, where $m(n) = \lfloor n - n^{3/4} \rfloor$. Let $K(\mathcal{P}_{m(n)}, r_0, \sigma)$ be the number of vertices in all components, which are neither isolated vertices nor the largest component. Let $f(n)$ be a function of n that satisfies (5). If $\pi r_0^2 \exp((2\eta^2\sigma^2)/(\alpha^2)) = ((f(n)+c)/n)$, then

$$\lim_{n\to\infty} \Pr\left\{K(\mathcal{P}_{m(n)}, r_0, \sigma) > 0\right\} = 0.$$

Proof: To prove the result, we make use of some results that were derived for the *Poisson random-connection model* in *continuum percolation* [38]. Let $\mathcal{H}_\lambda$ denote a homogeneous Poisson point process of intensity $\lambda > 0$ in $\Re^2$. For $s > 0$, define $B(s)$ to be a square of size $s \times s$ that was centered at the origin, i.e., $B(s) := [-s/2, s/2]^2$, and define $\mathcal{H}_{\lambda,s}$ to be the restriction of the homogeneous Poisson point process $\mathcal{H}_\lambda$ to the square $B(s)$, i.e., $\mathcal{H}_{\lambda,s} := \mathcal{H}_\lambda \cap B(s)$. It is also assumed that there is always a point at the origin.[4] In $\Re^2$, the Poisson random-connection model, which is denoted as $G(\mathcal{H}_{\lambda,s}, g)$, is a random graph with vertex set $\mathcal{H}_{\lambda,s}$ and *connection function* g. The connection function g is a function mapping from the positive reals into [0, 1], and any two vertices x_1 and x_2 of the point process $\mathcal{H}_{\lambda,s}$ are directly connected with probability $g(\|x_1 - x_2\|)$, where $\|\cdot\|$ denotes the Euclidean distance. The connection function g also has to satisfy the following conditions:

$$\begin{cases} g(x) = g(y) \text{ whenever } x = y \\ g(x) \leq g(y) \text{ whenever } x \geq y \\ 0 < \int_{\Re^2} g(x)dx < \infty. \end{cases} \tag{10}$$

The first restriction indicates that the propagation path is symmetric, the second restriction indicates that $g(x)$ must be a nonincreasing function of the distance x, and the third restriction avoids two trivial cases, i.e., $\int_{\Re^2} g(x)dx = 0$ and ∞. The two cases are not interesting, because in the first case, all nodes are isolated, and in the second case, all nodes are directly connected to each other [37], [38], [48], [49].

It has been proven that $G(\mathcal{H}_{\lambda,s}, g)$ has at most one infinite-order component for each $\lambda \geq 0$ [38, Th. 6.3]. In addition, when g satisfies (10), as $\lambda \to \infty$, a.a.s., the origin in the graph $G(\mathcal{H}_{\lambda,s}, g)$ either belongs to an infinite-order component or is isolated [38, Th. 6.4]. The implication of the aforementioned results is that, as $\lambda \to \infty$, the graph $G(\mathcal{H}_{\lambda,s}, g)$ a.a.s. only consists of a unique infinite-order component and a number of isolated vertices.

Let $s = (1/r_0)$, $\lambda = m(n)r_0^2$, and let

$$g(x) = \int_{10\alpha\log_{10}(x)}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}} dz.$$

It is clear that the graph $G(\mathcal{H}_{m(n)r_0^2,(1/r_0)}, g)$ is a Poisson random-connection model with vertex set $\mathcal{H}_{m(n)r_0^2,(1/r_0)}$ and connection function $g(x)$. In addition, $g(x)$ satisfies the conditions in (10). Because $m(n) = \lfloor n - n^{3/4} \rfloor$, $(\lfloor n - n^{3/4} \rfloor / n) \to 1$ as $n \to \infty$. Therefore

$$\lambda = m(n)r_0^2 \sim \frac{f(n) + c}{\pi \exp\left(\frac{2\eta^2\sigma^2}{\alpha^2}\right)} \to \infty \text{ as } n \to \infty.$$

Hence, as $n \to \infty$, the graph $G(\mathcal{H}_{m(n)r_0^2,(1/r_0)}, g)$ a.a.s. only consists of isolated vertices and an infinite-order component.

By space rescaling under the mapping $x \mapsto (1/r_0)x$ [20, Th. 9.17], it can be shown that the graph $G(\mathcal{H}_{m(n)r_0^2,(1/r_0)}, g(x))$ is similar to $G(\mathcal{H}_{m(n),1}, g(x/r_0))$, which means that they have the same statistical connectivity property. Therefore, the result (e.g., $\mathbb{P}_d(\mathcal{P}_{m(n)}, r_0, \sigma)$) for the graph $G(\mathcal{H}_{m(n)r_0^2,(1/r_0)}, g(x))$ also applies for $G(\mathcal{H}_{m(n),1}, g(x/r_0))$. $\mathbb{P}(x) = g(x/r_0)$, and $\mathcal{P}_{m(n)} = \mathcal{H}_{m(n),1}$; thus, the graph $G(\mathcal{P}_{m(n)}, r_0, \sigma)$ is the same as $G(\mathcal{H}_{m(n),1}, g(x/r_0))$ by

[4]Note that the distribution of points in a Poisson process does not depend on the assumption of the existence of a point at the origin (see *Slivnyak's theorem* [47]).

definition. Hence, as $n \to \infty$, the graph $G(\mathcal{P}_{m(n)}, r_0, \sigma)$ a.a.s. only consists of isolated vertices and an infinite-order component. Hence, $\Pr\{K(\mathcal{P}_{m(n)}, r_0, \sigma) > 0\} \to 0$ as $n \to \infty$. ■

Lemma 6: Consider $G(\mathcal{P}_{m(n)}, r_0, \sigma)$ in $\Re^2$, where $m(n) = \lfloor n - n^{3/4} \rfloor$. Let q be any fixed real number within (0, 1). Let c be any fixed real number. Let $f(n)$ be a function of n that satisfies (5). Ignore the boundary effect. If $\pi r_0^2 \exp((2\eta^2\sigma^2)/(\alpha^2)) = ((f(n) + c)/n)$, then

$$\lim_{n\to\infty} \Pr\left\{L\left(G\left(\mathcal{P}_{m(n)}, r_0, \sigma\right)\right) \geq qn\right\} = 1.$$

Proof: Let $N_{m(n)}$ be the number of points of $\mathcal{P}_{m(n)}$. Let $K(\mathcal{P}_{m(n)}, r_0, \sigma)$ denote the number of vertices in all components, which are neither isolated vertices nor the largest component in $G(\mathcal{P}_{m(n)}, r_0, \sigma)$. Let $I(\mathcal{P}_{m(n)}, r_0, \sigma)$ denote the number of isolated vertices in $G(\mathcal{P}_{m(n)}, r_0, \sigma)$. It is clear that $N_{m(n)} = L(G(\mathcal{P}_{m(n)}, r_0, \sigma)) + I(\mathcal{P}_{m(n)}, r_0, \sigma) + K(\mathcal{P}_{m(n)}, r_0, \sigma)$. Hence, by Lemma 5, we have

$$\begin{aligned}
&\Pr\left\{L\left(G\left(\mathcal{P}_{m(n)}, r_0, \sigma\right)\right) < qn\right\} \\
&\quad = \Pr\left\{N_{m(n)} - I\left(\mathcal{P}_{m(n)}, r_0, \sigma\right) - K\left(\mathcal{P}_{m(n)}, r_0, \sigma\right) < qn\right\} \\
&\quad = \Pr\left\{I\left(\mathcal{P}_{m(n)}, r_0, \sigma\right) > N_{m(n)} - qn\right\} + o(1) \\
&\qquad\qquad\qquad\qquad \text{as } n \to \infty. \qquad (11)
\end{aligned}$$

Define $I(j, r_0, \sigma)$ as the number of isolated vertices in $G(\mathcal{X}_j, r_0, \sigma)$, with $j \geq 0$. Then, we relate the uniform and Poisson distribution models in the next calculation. By (11) and Lemma 2, it can be obtained that as $n \to \infty$

$$\begin{aligned}
&\Pr\left\{L\left(G\left(\mathcal{P}_{m(n)}, r_0, \sigma\right)\right) < qn\right\} \\
&\quad = \Pr\left\{I\left(\mathcal{P}_{m(n)}, r_0, \sigma\right) > N_{m(n)} - qn\right\} + o(1) \\
&\quad = \sum_{j=0}^{\infty} \frac{(m(n))^j}{j!} e^{-m(n)} \Pr\left\{I(j, r_0, \sigma) > j - qn\right\} + o(1) \\
&\quad = \sum_{j \in J_n} \frac{(m(n))^j}{j!} e^{-m(n)} \Pr\left\{I(j, r_0, \sigma) > j - qn\right\} + o(1)
\end{aligned} \qquad (12)$$

where J_n is an integer set that was defined by (4).

By Lemma 4, it can be shown that, for any integer j that satisfies $j \in J_n$

$$\Pr\left\{I(j, r_0, \sigma) > j - qn\right\} = o(1), \qquad \text{as } n \to \infty. \qquad (13)$$

Substituting (13) into (12), we have

$$\Pr\left\{L\left(G\left(\mathcal{P}_{m(n)}, r_0, \sigma\right)\right) < qn\right\} = o(1), \qquad \text{as } n \to \infty.$$

Hence, the result immediately follows. ■

Now, we can prove Theorem 1 by de-Poissonizing Lemma 6.

Proof of Theorem 1: Let $m(n) = \lfloor n - n^{3/4} \rfloor$. Define $Y(\mathcal{P}_{m(n)}, r_0, \sigma)$ and $Y(\mathcal{X}_n, r_0, \sigma)$ as

$$\begin{aligned}
Y(\mathcal{P}_{m(n)}, r_0, \sigma) &:= \Pr\left\{L\left(G\left(\mathcal{P}_{m(n)}, r_0, \sigma\right)\right) < qn\right\} \\
Y(\mathcal{X}_n, r_0, \sigma) &:= \Pr\left\{L\left(G(\mathcal{X}_n, r_0, \sigma)\right) < qn\right\}.
\end{aligned}$$

Because $m(n) = \lfloor n - n^{3/4} \rfloor$, we have $Y(\mathcal{P}_{m(n)}, r_0, \sigma) \to 0$ as $n \to \infty$ by Lemma 6. Evidently, we need to show that $Y(\mathcal{X}_n, r_0, \sigma) \to 0$ as $n \to \infty$.

By Lemma 2, we have

$$\begin{aligned}
&Y\left(\mathcal{P}_{m(n)}, r_0, \sigma\right) \\
&\quad = \sum_{j=0}^{\infty} \frac{(m(n))^j}{j!} e^{-m(n)} Y(\mathcal{X}_j, r_0, \sigma) \\
&\quad = \sum_{j \in J_n} \frac{(m(n))^j}{j!} e^{-m(n)} Y(\mathcal{X}_j, r_0, \sigma) + o(1), \qquad \text{as } n \to \infty.
\end{aligned} \qquad (14)$$

Let $\mathcal{E}(\mathcal{X}_n, \mathcal{X}_j)$ denote the event that all nodes in $(\mathcal{X}_n \setminus \mathcal{X}_j)$ are isolated in $G(\mathcal{X}_n, r_0, \sigma)$. Then, for fixed r_0, σ, α, any fixed $q \in (0, 1)$, and any $j \in J_n$, it can be obtained that

$$\begin{aligned}
&Y(\mathcal{X}_n, r_0, \sigma) \\
&\quad \leq \Pr\left\{\mathcal{E}(\mathcal{X}_n, \mathcal{X}_j)\right\} + Y(\mathcal{X}_j, r_0, \sigma) \\
&\quad \sim \left[\left[1 - \pi r_0^2 \exp\left(\frac{2\eta^2\sigma^2}{\alpha^2}\right)\right]^{n-1}\right]^{n-j} + Y(\mathcal{X}_j, r_0, \sigma) \\
&\quad \leq \left(\frac{e^{-c}}{e^{f(n)}}\right)^{n-j} + Y(\mathcal{X}_j, r_0, \sigma) \\
&\quad = o(1) + Y(\mathcal{X}_j, r_0, \sigma), \qquad \text{as } n \to \infty. \qquad (15)
\end{aligned}$$

In the aforementioned derivation, Lemma 3 is used from the second to the third lines, and Lemma 1 is used from the third to the fourth lines. Substituting (15) into (14), it can be obtained that

$$\begin{aligned}
&Y\left(\mathcal{P}_{m(n)}, r_0, \sigma\right) \\
&\quad \geq \sum_{j \in J_n} \frac{(m(n))^j}{j!} e^{-m(n)} \left(Y(\mathcal{X}_n, r_0, \sigma) - o(1)\right) + o(1) \\
&\quad = Y(\mathcal{X}_n, r_0, \sigma) \sum_{j \in J_n} \frac{(m(n))^j}{j!} e^{-m(n)} + o(1) \\
&\quad = Y(\mathcal{X}_n, r_0, \sigma) + o(1), \qquad \text{as } n \to \infty. \qquad (16)
\end{aligned}$$

Because $Y(\mathcal{P}_{m(n)}, r_0, \sigma) = o(1)$ as $n \to \infty$ by Lemma 6, according to (16), we have

$$o(1) \geq Y(\mathcal{X}_n, r_0, \sigma) + o(1), \qquad \text{as } n \to \infty$$

which yields

$$\begin{aligned}
\Pr\left\{L\left(G(\mathcal{X}_n, r_0, \sigma)\right) \geq qn\right\} &= 1 - Y(\mathcal{X}_n, r_0, \sigma) \\
&= 1 - o(1), \qquad \text{as } n \to \infty.
\end{aligned}$$

The results immediately follow. ■

VI. Proof of Theorem 2

In this section, we will prove Theorem 2, which provides an asymptotic analytical lower bound on the minimum transmission power at which the probability of having a connected network asymptotically tends to one as $n \to \infty$. We first present Lemma 7, which will be used to prove Theorem 2.

Lemma 7: Let $\mathbb{P}_{iso}(\mathcal{X}_n, r_0, \sigma)$ denote the probability that an arbitrary node in $G(\mathcal{X}_n, r_0, \sigma)$ in $\Re^2$ is isolated. Let c be any fixed real number. Ignore the boundary effect. If $\pi r_0^2 \exp((2\eta^2\sigma^2)/(\alpha^2)) = (\log n + c/n)$, then

$$\mathbb{P}_{iso}(\mathcal{X}_n, r_0, \sigma) \sim \frac{e^{-c}}{n}, \quad \text{as } n \to \infty.$$

Proof: As shown in the proof of Lemma 4, ignoring the boundary effect and using Lemma 3, we have

$$\begin{aligned}\mathbb{P}_{iso}(\mathcal{X}_n, r_0, \sigma) &\sim [1 - \mathbb{P}(\mathcal{X}_n, r_0, \sigma)]^{n-1} \\ &\sim \left[1 - \pi r_0^2 \exp\left(\frac{2\eta^2\sigma^2}{\alpha^2}\right)\right]^{n-1}.\end{aligned}$$

Because $\pi r_0^2 \exp((2\eta^2\sigma^2)/(\alpha^2)) = ((\log n + c)/n) = ((\log n + c)/n) \to 0$ as $n \to \infty$, it can be obtained that

$$\begin{aligned}\mathbb{P}_{iso}\mathcal{X}_n, r_0, \sigma) &\sim \left[1 - \frac{\log n + c}{n}\right]^{n-1} \\ &\sim e^{-(\log n + c)} = \frac{e^{-c}}{n}, \qquad \text{as } n \to \infty.\end{aligned}$$

■

Now, we can prove Theorem 2 based on Lemma 7.

Proof of Theorem 2: Let $I(\mathcal{X}_n, r_0, \sigma)$ denote the number of isolated nodes in $G(\mathcal{X}_n, r_0, \sigma)$. It is clear that the probability that the network is disconnected is larger than or equal to the probability that the network has at least one isolated node, i.e.,

$$\begin{aligned}\mathbb{P}_d(\mathcal{X}_n, r_0, \sigma) &\geq \Pr\{I(\mathcal{X}_n, r_0, \sigma) \geq 1\} \\ &= 1 - \Pr\{I(\mathcal{X}_n, r_0, \sigma) = 0\}. \qquad (17)\end{aligned}$$

Because $n \gg 1$ and $r_0 \ll 1$, the event that a randomly selected node has i neighbors can be regarded almost independent of the event that another randomly selected node has j neighbors [5], [7], [9], [24], [37], [48], [50]. This independence assumption is based on the Palm theory [20, Th. 1.6], which captures a form of spatial ergodicity property that relates the probabilities that a given node has a certain degree. It has also been shown in [7], [24], and [50] that this independence assumption has provided a satisfactory level of approximation with large-enough n. Hence, ignoring the boundary effect and using Lemma 7, we have

$$\begin{aligned}\Pr\{I(\mathcal{X}_n, r_0, \sigma) = 0\} &= (1 - \mathbb{P}_{iso}(\mathcal{X}_n, r_0, \sigma))^n \\ &\sim \left(1 - \frac{e^{-c}}{n}\right)^n \\ &\sim \exp(-e^{-c}), \qquad \text{as } n \to \infty. \quad (18)\end{aligned}$$

By (17) and (18), the result follows. ■

VII. Arbitrary Channel Models

All our results in this paper are derived under the log-normal shadowing model. In addition to this wireless channel model and the unit disk communication model (obtainable by setting $\sigma = 0$ in the log-normal shadowing model), there also are other wireless channel models in the literature [21]. The derivation and analysis in this paper provide an efficient roadmap for extending these results to other channel models. In this section, we will explain this roadmap and how we can extend the results in this paper for other channel models.

We assume that the node distribution is the same (i.e., the vertex set is still $\mathcal{X}_n$) but that the wireless channel model is replaced by another channel model. Assume that $h(x)$ is the connection function that is associated with the new channel model, where $h(x)$ is a function of x mapping from positive reals into [0, 1], and x is the Euclidean distance between two nodes. Hence, any two nodes that were separated by a known distance x are directly connected with probability $h(x)$. In addition, assume that H is the probability that two randomly selected nodes in the network are directly connected. It is clear that H can be derived based on $h(x)$ and the node distribution. Different channel models will lead to different $h(x)$ and H. As an example, for the log-normal shadowing model, $h(x)$ is given by (2), and H is given by Lemma 3. If $h(x)$ satisfies the conditions given by (10), we can then obtain similar results comparable with the results (i.e., Theorems 1 and 2 and Corollary 1) in this paper in the same way as shown in Sections V and VI. The following discussion shows an example that is comparable with Theorem 1.

Example 1: Let $L(\mathcal{X}_n, h)$ denote the order of the largest component in the graph $G(\mathcal{X}_n, h)$, where $h = h(x)$ is the connection function that satisfies (10). Let H denote the probability that two randomly selected nodes in the graph $G(\mathcal{X}_n, h)$ are directly connected. Let q be any fixed real number within (0, 1). Let c be any fixed real number. Let $f(n)$ be a function of n that satisfies (5). Ignore the boundary effect. If $H = ((f(n) + c)/n)$, then

$$\lim_{n\to\infty} \Pr\{L(G(\mathcal{X}_n, h)) \geq qn\} = 1.$$

Based on the aforementioned analysis, we can obtain the following conclusion with regard to the ratio of the minimum transmission power that was required to have a giant component of order above qn ($q \in (0,1)$) to the minimum transmission power that was required to have a connected network; different channel models may result in different quantitative changes in the ratio but do not change the qualitative nature of the ratio, i.e., the ratio will tend to zero as $n \to \infty$, although the speed of the decrease may slightly change under different channel models. In other words, the transmission power that was required to have a giant component of order above qn is vanishingly small compared with the transmission power that was required to have a connected network for all channel models that satisfy the conditions given by (10).

Remark: The key ingredient for such an extension is that the connection function $h(x)$ associated with the new channel model *must* satisfy the rotational symmetry, monotonicity, and integral boundedness conditions given by (10).

VIII. Conclusion

In this paper, we have investigated the giant component in 2-D wireless multihop networks by employing the log-normal shadowing channel model, which is more realistic than the unit disk communication model that was used in previous papers. We have derived an asymptotic analytical upper bound on the minimum transmission power to have a giant component of order above qn (see Theorem 1) and have also derived an asymptotic analytical lower bound on the minimum transmission power to have a connected network (see Theorem 2). Based on these two results, we have further shown that the minimum transmission power that was required to have a giant component of order above qn is vanishingly small compared with the minimum transmission power that was required to have a connected network (see Corollary 1). This result means that significant energy savings can be achieved if we require only most nodes (e.g., 95%) to be connected to the giant component rather than requiring all nodes to be connected, particularly for a large-scale network. We have also provided a roadmap for extending our results, which we have obtained under the log-normal shadowing model, to other wireless channel models.

There are several directions for future work. First, we will investigate the giant component problem by taking into account the boundary effect, which has been ignored in this paper to avoid complexity in the analysis. Second, our results are derived for asymptotically infinite n, and one will also be interested in the results for small values of n for practical purposes; thus, it is also important to investigate this problem for small values of n. Last, all the results are derived for static networks; it will also be of interest and is important to analyze the problems for mobile networks.

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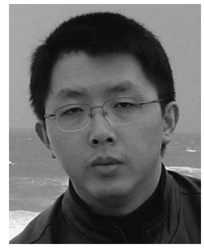

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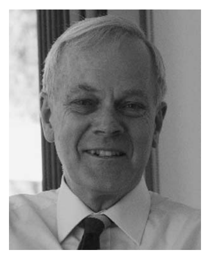

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