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Interference Minimization in 5G Heterogeneous Networks

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Abstract In this paper, we focus on one of the representative 5G network scenarios, namely multi-tier heterogeneous cellular networks. User association is investigated in order to reduce the down-link co-channel interference. Firstly, in order to analyze the multi-tier heterogeneous cellular networks where the base stations in different tiers usually adopt different transmission powers, we propose a Transmission Power Normalization Model (TPNM), which is able to convert a multi-tier cellular network into a single-tier network, such that all base stations have the same normalized transmission power. Then using TPNM, the signal and interference received at any point in the complex multi-tier environment can be analyzed by considering the same point in the equivalent single-tier cellular network model, thus significantly simplifying the analysis. On this basis, we propose a new user association scheme in heterogeneous cellular networks, where the base station that leads to the smallest interference to other co-channel mobile stations is chosen from a set of candidate base stations that satisfy the quality-of-service (QoS) constraint for an intended mobile station. Numerical results show that the proposed user association scheme is able to significantly reduce the down-link interference compared with existing schemes while maintaining a reasonably good QoS.

Keywords Heterogeneous cellular networks · user association · performance analysis model · interference management

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1 Introduction

A heterogeneous cellular network (HCN) usually consists of multiple tiers including a macrocell tier and probably some small cell tiers, e.g., picocell tier, femtocell tier and so on [1]. In general, there are three channel allocation strategies among tiers, namely orthogonal deployment, co-channel deployment, and partially shared deployment [3]. In order to improve the spectral efficiency to match the ever growing demand for high data rate nowadays and future, co-channel deployment among tiers and spatial frequency reuse are widely employed in HCNs. In the small cell tier, since base stations (BSs) are often deployed in an unplanned manner, it causes more serious co-channel interference in heterogeneous networks than that in conventional single-tier cellular networks. In view of the severe co-channel interference under both intra-tier and inter-tier situations [12], interference management is very important in a HCN [7].

User association, also called cell association or BS association, is one of the important approaches to performing interference management as well as to improving the spectral efficiency and energy efficiency [13]. Fooladivanda *et al.* proposed a unified static framework to study the interplay between user association and resource allocation in HCNs [3]. Ghimire *et al.* formulated a flow-based framework for the joint optimization of resource allocation, transmission coordination, and user association in a heterogeneous network comprising of a macro BS and a number of pico BSs and/or relay nodes [5], where the performance of different combinations of resource allocation schemes and transmission coordination mechanisms was characterized. Jin *et al.* proposed a marginal utility based user association algorithm to transform the combinatorial optimization problem into a network-wide utility maximization problem [8]. Jo *et al.* developed a tractable framework for signal-to-interference-plus-noise ratio (SINR) analysis in downlink HCNs with flexible cell association policies [6]. Madan *et al.* described new paradigms for the design and operation of HCNs, where cell splitting, cell range expansion, semi-static resource negotiation on third-party backhaul connections, and fast dynamic interference management for quality-of-service (QoS) via over-the-air signaling were investigated [10].

In HCNs, an active mobile station (MS) needs to associate itself with a particular cell, which belongs to one of the tiers in a multi-tier network. Conventionally, a MS is associated with the nearest BS or the BS that provides the highest received SINR. However, these MS association schemes do not consider the possible co-channel interference caused to other active MSs. Motivated by this, in this paper using stochastic geometry methods [2], we propose a MS association scheme in multi-tier networks that is able to significantly reduce the down-link co-channel interference while guaranteeing a predefined QoS of mobile users in HCNs with open-access small cell. Consider the interference in up-link is not always minimized when we minimize the interference in down-link by a user association scheme, we will not investigate the issue on interference in up-link in this paper. The contributions of this paper are:

1. A Transmission Power Normalization Model (TPNM) for analyzing the performance of multi-tier HCNs is proposed, which significantly simplifies the analysis of the performance of multi-tier HCNs.
2. Based on TPNM, a new user association scheme is proposed to minimize the down-link co-channel interference, which can be used in both conventional single-tier cellular networks and multi-tier HCNs.
3. Extensive simulations are conducted, the results demonstrate that the proposed scheme can significantly reduce the down-link interference under the constraint that predefined QoS requirements are satisfied.

The rest of the paper is organized as follows. Section 2 describes the system model. Section 3 introduces the proposed TPNM for performance analysis in multi-tier HCNs. Based on TPNM, we proceed to propose a new user association scheme to minimize the down-link co-channel interference in section 4. Section 5 shows the numerical results for the performance of the proposed user association scheme. In section 6, we conclude the paper.

2 System model

We consider a heterogeneous cellular multi-tier network that is composed of K-tier networks where $K \in \mathbb{N}$ with only a single BS located winthin each cell of the multi-tier networks. The transmission powers at the BSs of the k-th tier network are assumed to be equal and denoted as P_k. We assume that the distribution of the BSs in the k-th tier network follows a homogeneous Poisson point process Φ_k^{BS} with intensity λ_k^{BS}. Assuming that the multiple cells of different tiers are overlaid in the same area geographically, then the distribution of the BSs in multi-tier HCNs is governed by a Poisson point process $\Phi^{\mathrm{BS}} = \bigcup_{k=1}^{K} \Phi_k^{\mathrm{BS}}$ with intensity $\lambda^{\mathrm{BS}} = \sum_{k=1}^{K} \lambda_k^{\mathrm{BS}}$.

Furthermore, we assume that the distribution of active MSs which are associated with the BSs in the k-th tier network follows a homogeneous Poisson process Φ_k^{MS} of intensity λ_k^{MS}. Thus the distribution of all MSs in multi-tier HCNs is also governed by a Poisson point process $\Phi^{\mathrm{MS}} = \bigcup_{k=1}^{K} \Phi_k^{\mathrm{MS}}$ with intensity $\lambda^{\mathrm{MS}} = \sum_{k=1}^{K} \lambda_k^{\mathrm{MS}}$.

This paper focuses on the down-links in multi-tier HCNs, where all BSs reuse the same frequency that is divided into orthogonal channels. A BS allocates different orthogonal channels to the MSs in a cell. Under such circumstances, there is no intra-cell interference. However, due to the frequency reuse across cells, there may exist severe inter-cell co-channel interference in multi-tier HCNs if the same sub-channel is occupied in different cells [11]. For example, given that the BSs assign the channels randomly and independently, at a particular time instant, only a fraction of the BSs, denoted by Poisson point process $\Phi^{\mathrm{N\_BS}}$ of intensity $\lambda^{\mathrm{N\_BS}}$, are using a specific channel C_n simultaneously to transmit to the corresponding MSs, denoted by Poisson point process $\Phi^{\mathrm{N\_MS}}$ of intensity $\lambda^{\mathrm{N\_MS}} = \lambda^{\mathrm{N\_BS}}$, where the BSs in $\Phi^{\mathrm{N\_BS}}$ and the MSs in $\Phi^{\mathrm{N\_MS}}$ are communication pairs.

Assuming BSs assign down-link channels to the MSs associated with them randomly, then the MSs using the same channel C_n, i.e. $\Phi^{\mathrm{N\_MS}}$, can be considered to follow a homogeneous Poisson point process [15], which is thinned from point process Φ^{MS}. Then we define $\Phi^{\mathrm{N\_MS}}$ as an interfering set, in which MSs are interfered by the

BSs that are transmitting to other MSs in the set because they use the same channel C_n.

For ease of exposition, only path loss effect is considered in the wireless channel models. Without loss of generality, we consider a given BS x and a desired MS y. Then the desired signal power P_{xy} received at y is expressed as

$$P_{xy} = P_x \, l\left(x - y\right), \tag{1}$$

where P_x denotes the transmission power of the BS and $l\left(\cdot\right) = \left\|\cdot\right\|^{-\alpha}$ denotes the path loss in wireless channels where α is the path loss exponent.

In this paper, we focus on the interference-limited scenario. When a MS y is associated with a BS x, the signal-to-interference ratio (SIR) at y is given as

$$\mathrm{SIR}\left(x, y\right) = \frac{P_{xy}}{I_y} = \frac{P_x \, l\left(x - y\right)}{\sum_{x_i \in \Phi^{\mathrm{N\_BS}} \setminus \{x\}} P_{x_i} \, l\left(x_i - y\right)}, \tag{2}$$

where I_y denotes the interference received from the BSs in $\Phi^{\mathrm{N\_BS}}$ except x_i.

In view of the severe co-channel interference, we consider a user association scheme where the MS $y \in \Phi^{\mathrm{N\_MS}}$ chooses a BS $x \in \Phi^{\mathrm{BS}}$ to associate with, and at the same time the interference from x to other MSs $\Phi^{\mathrm{N\_MS}} \setminus \{y\}$ is minimized.

In order to minimize the interference caused by the chosen BS x to other co-channel MSs, for ease of analysis, we consider a MS $z \in \Phi^{\mathrm{N\_MS}} \setminus \{y\}$ that receives the most severe interference I_{xz} from BS x [9]. Then the minimization of the interference seen at z probably implies a minimization of the co-channel interference.

On the other hand, to satisfy a reasonable QoS constraint, it is assumed that the distance between the specific MS y and the corresponding BS $x \in \Phi^{\mathrm{BS}}$ should be no more than the distance between y and any BS $\forall x_i \in \Phi^{\mathrm{N\_BS}}$ transmitting in the same channel. In other words, we intend to choose a suitable BS for y such that the co-channel interference caused to other MSs is minimized, under the constraint the QoS of the MS y is satisfied. If in Φ^{BS} there is no BS satisfies this constraint, then MS y will try to search another channel.

3 TPNM of HCN

3.1 Definition of TPNM

Different from single-tier homogeneous cellular networks where all the BSs transmit signal using the same power, in multi-tier HCNs, the BSs of different tiers have different transmission powers and follow different distributions geographically.

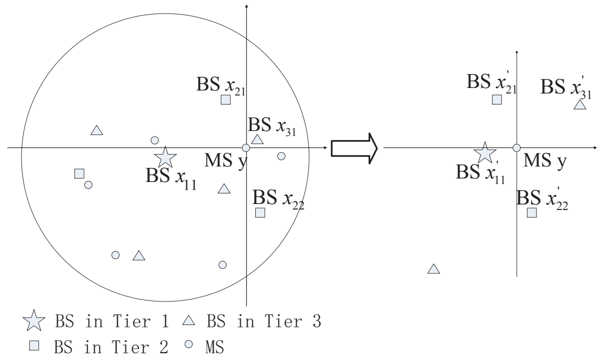

Fig. 1 By TPNM, each tier is scaled by using the location of a specific MS y as the scaling center.

In order to analyze the multi-tier HCN, we propose a TPNM in this paper, which is able to convert a multi-tier HCN to a virtual single-tier cellular network by first scaling each tier according to its corresponding transmission power and path-loss effect, and then combining the different tiers into a single-tier cellular network, such that all BSs have the same normalized transmission power and the signal power and interference received at a specific MS from the BSs in the virtual single-tier cellular network are exactly the same as those received from the BSs in the original multi-tier cellular network.

As an example, consider a 3-tier HCN shown in Fig. 1. There are BSs in various tiers including BS x_{11} in tier 1, BS x_{21} and x_{22} in tier 2, and BS x_{31} in tier 3, with different transmission powers. MS y receives the desired signal from the associated BS and interference from the other BSs. For ease of analysis, we set the location of the MS y as the origin and scale each tier by using different factors such that virtual BSs x'_{11}, x'_{21}, x'_{22} and x'_{31} with the same normalized power 1 are obtained, and the received signal/interference powers at y from these virtual BSs are exactly the same as those received from the original BSs, e.g., the power received at MS y from BS x_{11} before scaling is exactly the same as that from virtual BS x'_{11} after scaling.

In a K-tier HCN, the BSs in tier k, $k \in \{1, 2, \ldots, K\}$, have transmission power P_k and follow a Poisson point process Φ_k^{BS} of intensity λ_k^{BS}. Without loss of generality, we consider a MS y located at origin o, then the received signal power P_{xy} at MS y from a BS $x \in \Phi_k$ is given as

$$\begin{aligned} P_{xy} &= P_k \, l\left(x - o\right) = P_k \, l\left(x\right) = 1 \cdot \left(P_k^{-\frac{1}{\alpha}} \left\|x\right\|\right)^{-\alpha} \\ &= 1 \cdot \left\|P_k^{-\frac{1}{\alpha}} \cdot x\right\|^{-\alpha} = 1 \cdot l\left(P_k^{-\frac{1}{\alpha}} \cdot x\right), \end{aligned} \tag{3}$$

where 1 is the normalized transmission power and $l(x - o)$ is the path loss function from x to y.

From Eq. (3), it is observed that the signal power received at y from x is equal to that received from a virtual BS $x' = P_k^{-\frac{1}{\alpha}} \cdot x$ with transmission power 1 and located at $P_k^{-\frac{1}{\alpha}} \cdot x$. Following this, the Poisson point process Φ_k^{BS} can be scaled to a point process $\Phi_k^{\mathrm{BS}\prime} = P_k^{-\frac{1}{\alpha}} \cdot \Phi_k^{\mathrm{BS}}$ of intensity $\lambda_k^{\mathrm{BS}\prime} = \left(\frac{1}{p_k^{-\frac{1}{\alpha}}}\right)^2 \lambda_k^{\mathrm{BS}} = P_k^{\frac{2}{\alpha}} \lambda_k^{\mathrm{BS}}$, in which all the virtual BSs have the same transmission power 1 and produce the same received signal power (or interference power) at the MS y as the original BSs in Φ_k^{BS}.

From the above analysis, we scale the Poisson point processes in all K tiers to normalize the BSs' transmission powers to 1, and then combine them into a single Poisson point process

$$\Phi^{\mathrm{BS}\prime} = \bigcup_{k=1}^{K} P_k^{-\frac{1}{\alpha}} \cdot \Phi_k^{\mathrm{BS}}, \tag{4}$$

which is of intensity $\lambda^{\mathrm{BS}\prime} = \sum_{k=1}^{K} P_k^{\frac{2}{\alpha}} \lambda_k^{\mathrm{BS}}$.

3.2 Received signal power based on TPNM

In this section, we give an example to demonstrate the advantage of using TPNM by considering analysis of cell association where a MS always associate with the BS delivering the highest received signal strength. We first analyze the case without TPNM, then we present the analysis by using TPNM.

3.2.1 The received signal power without TPNM

Consider a specific receiving MS y, without loss of generality, we place it at the origin $o \in \mathbb{R}^2$. Then the BS

$$x = \arg \max_{x_i \in \Phi^{\mathrm{BS}}} P_{x_i} \left\| x_i \right\|^{-\alpha}, \tag{5}$$

which can produce the highest received signal power at y, is selected to associate with y.

Assuming there are K tiers in the network with corresponding transmission power $P_k, k \in \{1, 2, \ldots, K\}$, we find the nearest BS x_k from each tier Φ_k^{BS} to y, i.e.,

$$x_k = \arg \min_{x_{kj} \in \Phi_k^{\mathrm{BS}}} \left\| x_{kj} \right\|. \tag{6}$$

According to Slivnyak theorem [14], $\Phi_k^{\mathrm{BS}} \cup \{o\}$ has the same properties as the Poisson point process Φ_k^{BS}, so the distance $R_k \triangleq \|x_k - y\| = \|x_k - o\| = \|x_k\|$ between BS x_k and MS y satisfies the following probability density function (PDF)

$$f_{R_k}(r_k) = 2\pi\lambda_k^{\mathrm{BS}} r_k \cdot \mathrm{e}^{-\lambda_k^{\mathrm{BS}} \pi r_k^2}. \tag{7}$$

Then the signal power received at MS y, i.e., $P_{x_k y} = P_k R_k^{-\alpha}$, has the following PDF

$$\begin{aligned} f_{P_{x_k y}(p_{x_k y})} &= f_{R_k}(r_k) \cdot \left| \left(\left(\frac{P_k}{p_{x_k y}} \right)^{\frac{1}{\alpha}} \right)' \right| \\ &= \frac{2\pi\lambda_k^{\mathrm{BS}}}{\alpha p_{x_k y}} \left(\frac{P_k}{P_{x_k y}} \right)^{\frac{2}{\alpha}} \mathrm{e}^{-\pi\lambda_k^{\mathrm{BS}} \left(\frac{P_k}{p_{x_k y}} \right)^{\frac{2}{\alpha}}}. \end{aligned} \tag{8}$$

Defining $D_{x_k y} \triangleq P_{x_k y}^{-\frac{1}{\alpha}} = P_k^{-\frac{1}{\alpha}} R_k$, we obtain the PDF of $D_{x_k y}$ as follow:

$$\begin{aligned} f_{D_{x_k y}}(d_{x_k y}) &= f_{P_{x_k y}}(P_{x_k y}) \cdot \left| \left(d_{x_k y}^{-\alpha} \right)' \right| \\ &= 2\pi\lambda_k^{\mathrm{BS}} P_k^{\frac{2}{\alpha}} d_{x_k y} \cdot \mathrm{e}^{-\pi\lambda_k^{\mathrm{BS}} P_k^{\frac{2}{\alpha}} d_{x_k y}^2}, \end{aligned} \tag{9}$$

and the corresponding cumulative distribution function (CDF) is derived as

$$\begin{aligned} F_{D_{x_k y}}(d_{x_k y}) &= \int_{-\infty}^{d_{x_k y}} f_{D_{x_k y}}(d_{x_k y}) \, \mathrm{d}d_{x_k y} \\ &= 1 - \mathrm{e}^{-\pi\lambda_k^{\mathrm{BS}} P_k^{\frac{2}{\alpha}} d_{x_k y}^2}. \end{aligned} \tag{10}$$

The BS that MS y is associated with should have the largest $P_{x_k y}$, so it should have the smallest $D_{x_k y}$ as well. Denote the smallest $D_{x_k y}$ by D_{xy}, and the largest $P_{x_k y}$ by P_{xy}, we obtain

$$P_{xy} = \max_{k \in \{1,2,\ldots,K\}} P_{x_k y}, \tag{11}$$

and

$$D_{xy} = \min_{k \in \{1,2,\ldots,K\}} D_{x_k y}. \tag{12}$$

Since random variables $D_{x_k y}$, $k \in \{1, 2, \ldots, K\}$ are mutually independent, then the CDF of D_{xy} can be derived as

$$\begin{aligned} F_{D_{xy}}(d_{xy}) &= 1 - \prod_{k=1}^{K} \left(1 - \left(1 - \mathrm{e}^{-\pi\lambda_k^{\mathrm{BS}} P_k^{\frac{2}{\alpha}} d_{x_k y}^2} \right) \right) \\ &= 1 - \mathrm{e}^{-\pi d_{x_k y}^2 \sum_{k=1}^{K} \lambda_k^{\mathrm{BS}} P_k^{\frac{2}{\alpha}}}, \end{aligned} \tag{13}$$

and the PDF of D_{xy} can be derived as

$$f_{D_{xy}}(d_{xy}) = (2\pi) d_{xy}^2 \sum_{k=1}^{K} \lambda_k^{\mathrm{BS}} P_k^{\frac{2}{\alpha}} \cdot \mathrm{e}^{-\pi d_{x_k y}^2 \sum_{k=1}^{K} \lambda_k^{\mathrm{BS}} P_k^{\frac{2}{\alpha}}}. \tag{14}$$

Because $D_{xy} = P_{xy}^{-\frac{1}{\alpha}}$, we have

$$\begin{aligned} F_{P_{xy}}(P_{xy}) &= F_{D_{xy}}\left(P_{xy}^{-\frac{1}{\alpha}} \right) \\ &= 1 - \mathrm{e}^{-\pi P_{xy}^{-\frac{2}{\alpha}} \sum_{k=1}^{K} \lambda_k^{\mathrm{BS}} P_k^{\frac{2}{\alpha}}}. \end{aligned} \tag{15}$$

3.2.2 The received signal power with TPNM

By using TPNM, we scale each tier with factor $P_k^{-\frac{1}{\alpha}}$, and then combine them to a virtual Poisson point process $\Phi^{\mathrm{BS}\prime}$ of intensity $\lambda^{\mathrm{BS}\prime} = \sum_{k=1}^{K} P_k^{\frac{2}{\alpha}} \lambda_k^{\mathrm{BS}}$. Denote the distance between MS y and the nearest BS $x \in \Phi^{\mathrm{BS}\prime}$ by R, which has the following CDF

$$F_R(r) = 1 - \mathrm{e}^{-\lambda^{\mathrm{BS}\prime}\pi r^2} = 1 - \mathrm{e}^{-\sum_{k=1}^{K} P_k^{\frac{2}{\alpha}} \lambda_k^{\mathrm{BS}} \pi r^2}. \quad (16)$$

Because $P_{xy} = 1 \cdot R^{-\alpha}$ and thus $R = P_{xy}^{-\frac{1}{\alpha}}$, we have

$$\begin{aligned} F_{P_{xy}}(p_{xy}) &= 1 - \mathrm{e}^{-\sum_{k=1}^{K} P_k^{\frac{2}{\alpha}} \lambda_k^{\mathrm{BS}} \pi \left(p_{xy}^{-\frac{1}{\alpha}}\right)^2} \\ &= 1 - \mathrm{e}^{-\pi p_{xy}^{-\frac{2}{\alpha}} \sum_{k=1}^{K} \lambda_k^{\mathrm{BS}} P_k^{\frac{2}{\alpha}}}. \end{aligned} \quad (17)$$

Since Eq. (17) is of the same form as Eq. (15), TPNM can be used to analyze the received signal and interference powers at an arbitrary MS with path loss effect, which significantly simplifies the derivations.

4 Interference minimized user association scheme

4.1 Interference modeling of HCNs

In a multi-tier HCN with different transmission powers across tiers, each active MS chooses the BS that produces the highest received SINR to associate with.

Consider an arbitrary MS y, by using TPNM, the multi-tier HCN can be transformed to a virtual single-tier cellular network $\Phi^{\mathrm{BS}\prime} = \bigcup_{k=1}^{K} P_k^{-\frac{1}{\alpha}} \cdot \Phi_k^{\mathrm{BS}}$ of intensity $\lambda^{\mathrm{BS}\prime} = \sum_{k=1}^{K} P_k^{\frac{2}{\alpha}} \lambda_k^{\mathrm{BS}}$. Then the average fraction of users that are served by tier k in open access is given as

$$\bar{N}_k = \frac{\lambda_k^{\mathrm{BS}\prime}}{\lambda^{\mathrm{BS}\prime}} = \frac{\lambda_k^{\mathrm{BS}} P_k^{\frac{2}{\alpha}}}{\sum_{k=1}^{K} \lambda_k^{\mathrm{BS}} P_k^{\frac{2}{\alpha}}}. \quad (18)$$

Then in tier k, the nearest BS x_k is selected associate with MS y. To evaluate the interference caused by x_k to other co-channel MSs, we consider the nearest MS to x_k other than y, i.e.,

$$z_k = \arg \min_{z_i \in \Phi^{\mathrm{N\_MS}} \setminus \{y\}} \|x_k - z_i\|, \quad (19)$$

then the received interference at z_k from x_k is given as

$$I_{x_k z_k} = P_k\, l(R_{x_k z_k}) = P_k \|x_k - z_k\|^{-\alpha}, \quad (20)$$

where the distance $R_{x_k z_k} = \|x_k - z_k\|$ between x_k and z_k follows the following CDF and PDF respectively:

$$F_{R_{x_k z_k}}(r_{x_k z_k}) = 1 - \mathrm{e}^{-\lambda^{\mathrm{N\_MS}} \pi r_{x_k z_k}^2}, \quad (21)$$

$$f_{R_{x_k z_k}}(r_{x_k z_k}) = 2\pi \lambda^{\mathrm{N\_MS}} r_{x_k z_k} \cdot \mathrm{e}^{-\pi \lambda^{\mathrm{N\_MS}} r_{x_k z_k}^2}. \quad (22)$$

According to (18), we consider the probability that the BS x, which is serving MS y, belongs to the k-th tier also as $\bar{N}_k$. Then the expectation of the interference received at MS z from BS x can be derived as

$$\begin{aligned} \mathbb{E}(I_{xz}) &= \sum_{k=1}^{K} \bar{N}_k \int_0^\infty f_{R_{x_k z_k}}(r_{x_k z_k}) \cdot P_k\, l(r_{x_k z_k})\, \mathrm{d}r_{x_k z_k} \\ &= \frac{\sum_{k=1}^{K} \lambda_k^{\mathrm{BS}} P_k^{\frac{2}{\alpha}+1}}{\sum_{k=1}^{K} \lambda_k^{\mathrm{BS}} P_k^{\frac{2}{\alpha}}} \cdot \int_0^\infty l(r_{x_k z_k})\, f_{R_{x_k z_k}}(r_{x_k z_k})\, \mathrm{d}r_{x_k z_k}. \end{aligned} \quad (23)$$

4.2 Interference minimized user association scheme

In the proposed user association scheme, consider a specific MS, the BS x_{opt} that generates the largest received SIR at this MS is selected under the constraint on the predefined QoS.

Consider an arbitrary MS y, we first transform the multi-tier HCN to a virtual single-tier cellular network $\Phi^{\mathrm{BS}\prime} = \bigcup_{k=1}^{K} P_k^{-\frac{1}{\alpha}} \cdot \Phi_k^{\mathrm{BS}}$ of intensity $\lambda^{\mathrm{BS}\prime} = \sum_{k=1}^{K} P_k^{\frac{2}{\alpha}} \lambda_k^{\mathrm{BS}}$ by TPNM. Then we have the interfering set that transmit simultaneously in channel C_n as $\Phi^{\mathrm{N\_BS}\prime} \subset \Phi^{\mathrm{BS}\prime}$ of intensity $\lambda^{\mathrm{N\_BS}\prime}$, which is transformed from $\Phi^{\mathrm{N\_BS}}$ by TPNM as well.

To satisfy the QoS constraint, not all BSs in $\Phi^{\mathrm{BS}\prime}$ can be chosen to communicate with the MS y, we denote the subset of BSs that are allowed to communicate with y by $T_y \subset \Phi^{\mathrm{BS}\prime}$. Then the distance between y and each BS in T_y is no more than the distance between y and any other transmitting-in-the-same-channel BS, i.e.,

$$\begin{aligned} T_y = \big\{ x_i' : \|x_i' - y\| \le \|x_j' - y\| \\ , x_i' \in \Phi^{\mathrm{BS}\prime}, \forall x_j' \in \Phi^{\mathrm{N\_BS}\prime} \big\}. \end{aligned} \quad (24)$$

We denote the number of BSs in subset T_y by a random variable N_{T_y}. N_{T_y} is the number of points from $\Phi^{\mathrm{BS}\prime}$ in the void ball $V = b\left(y, R^{\mathrm{N\_BS}\prime}\right)$ of $\Phi^{\mathrm{N\_BS}\prime}$, where $R^{\mathrm{N\_BS}\prime}$ is the void distance of $\Phi^{\mathrm{N\_BS}\prime}$, whose PDF is [14]

$$f_{R^{\mathrm{N\_BS}\prime}}\left(r^{\mathrm{N\_BS}\prime}\right) = 2\pi \lambda^{\mathrm{N\_BS}\prime} r^{\mathrm{N\_BS}\prime} \cdot \mathrm{e}^{-\pi \lambda^{\mathrm{N\_BS}\prime} \left(r^{\mathrm{N\_BS}\prime}\right)^2}. \quad (25)$$

Then an estimated value $\bar{N}_{T_y}$ of N_{T_y} is given as

$$\begin{aligned} \bar{N}_{T_y} &\triangleq \mathbb{E}\left(N_{T_y}\right) = \lambda^{\mathrm{BS}\prime} \cdot A(V) \\ &= \lambda^{\mathrm{BS}\prime} \int_0^\infty \pi \left(r^{\mathrm{N\_BS}\prime}\right)^2 \cdot f_{R^{\mathrm{N\_BS}\prime}}\left(r^{\mathrm{N\_BS}\prime}\right) \mathrm{d}r^{\mathrm{N\_BS}\prime} \\ &= \frac{\lambda^{\mathrm{BS}\prime}}{\lambda^{\mathrm{N\_BS}\prime}}, \end{aligned} \quad (26)$$

where $A(V)$ denotes the area of V.

We assume that the proportion of the transmitting BSs in each tier is the same. Then according to (26), we obtain

$$\bar{N}_{T_y} = \frac{\lambda^{\mathrm{BS}\prime}}{\lambda^{\mathrm{N\_BS}\prime}} = \frac{\lambda^{\mathrm{BS}\prime}}{\lambda^{\mathrm{N\_BS}} \cdot \frac{\lambda^{\mathrm{BS}\prime}}{\lambda^{\mathrm{BS}}}} = \frac{\lambda^{\mathrm{BS}}}{\lambda^{\mathrm{N\_BS}}}. \quad (27)$$

According to (18), the average fraction of users served by tier k in open access can thus be derived as

$$\bar{N}_k = \frac{\lambda_k^{\mathrm{BS}} P_k^{\frac{2}{\alpha}}}{\sum_{k=1}^{K} \lambda_k^{\mathrm{BS}} P_k^{\frac{2}{\alpha}}}. \quad (28)$$

Then the BS that satisfies

$$x_{\mathrm{opt}} = \arg \min_{x_i' \in T_y} \max_{z_j' \in \Phi^{\mathrm{N\_MS}\prime} \setminus \{y\}} 1 \cdot \left\| x_i' - z_j' \right\|^{-\alpha} \quad (29)$$

is selected to associate with y. For the co-channel MS that receives the largest interference from x_{opt}, i.e., z_{opt}, we have

$$z_{\mathrm{opt}} = \arg \min_{z_i' \in \Phi^{\mathrm{N\_MS}\prime} \setminus \{y\}} \left\| x_{\mathrm{opt}} - z_i' \right\|. \quad (30)$$

Denote the distance between x_{opt} and z_{opt} by $R_{\mathrm{opt}} = \|x_{opt} - z_{\mathrm{opt}}\|$, then the CDF and PDF of R_{opt} are derived as

$$F_{R_{\mathrm{opt}}}(r_{\mathrm{opt}}) = \left(1 - \mathrm{e}^{-\lambda^{\mathrm{N\_MS}} \pi r_{\mathrm{opt}}^2}\right)^{\bar{N}_{T_y}}, \quad (31)$$

$$f_{R_{\mathrm{opt}}}(r_{\mathrm{opt}}) = 2\pi \bar{N}_{T_y} \lambda^{\mathrm{N\_MS}} r_{\mathrm{opt}} \cdot \mathrm{e}^{-\lambda^{\mathrm{N\_MS}} \pi r_{\mathrm{opt}}^2} \cdot \left(1 - \mathrm{e}^{-\lambda^{\mathrm{N\_MS}} \pi r_{\mathrm{opt}}^2}\right)^{\bar{N}_{T_y} - 1}, \quad (32)$$

respectively.

And then the expectation of the interference received at MS z_{opt} from BS x_{opt} can be similarly derived as

$$\mathbb{E}\left(I_{x z_{\mathrm{opt}}}\right) = \int_0^{\infty} f_{R_{\mathrm{opt}}}(r_{\mathrm{opt}}) \cdot \sum_{k=1}^{K} \bar{N}_k P_k \cdot l(r_{\mathrm{opt}})\, \mathrm{d}r_{\mathrm{opt}} = \frac{\sum_{k=1}^{K} \lambda_k P_k^{\frac{2}{\alpha}+1}}{\sum_{k=1}^{K} \lambda_k P_k^{\frac{2}{\alpha}}} \cdot \int_0^{\infty} f_{R_{\mathrm{opt}}}(r_{\mathrm{opt}}) \cdot l(r_{\mathrm{opt}})\, \mathrm{d}r_{\mathrm{opt}}. \quad (33)$$

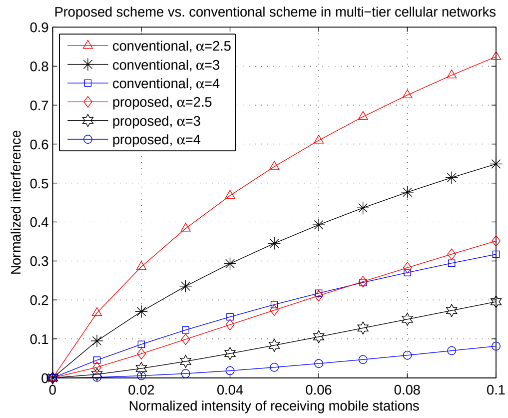

Fig. 2 Interference in a 3-tier cellular network where $P_k = \{10, 1, 0.1\}$, $\lambda_k^{\mathrm{BS}} = \{0.01, 0.1, 1\}$ and $\lambda^{\mathrm{BS}}/\lambda^{\mathrm{N\_BS}} = 3$.

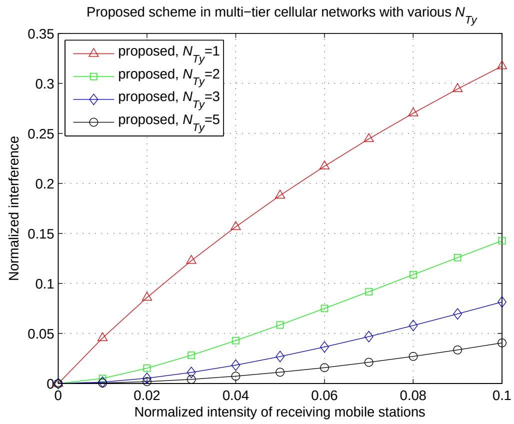

Fig. 3 Interference in a 3-tier cellular network where $P_k = \{10, 1, 0.1\}$, $\lambda_k^{\mathrm{BS}} = \{0.01, 0.1.1\}$ and $\alpha = 4$.

5 Numerical results

In this section, we present the analytical results of the proposed BS association scheme and compare it to the conventional scheme that is subject to severe co-channel interference. To avoid the singularity of path loss function $l(\cdot)$ in (23) and (33), we use $l(r) = (1 + r^{\alpha})^{-1}$ [4] instead of $l(r) = r^{-\alpha}$ in deriving the analytical results.

In Fig. 2, the interference in a 3-tier cellular network is demonstrated and compared between the proposed scheme and the conventional scheme. The BS transmission powers in tier 1, 2 and 3 are 10, 1 and 0.1 respectively, and the intensities of BSs in tier 1, 2 and 3 are 0.01, 0.1 and 1 respectively. The result indicates that the proposed scheme is effective to reduce the interference in multi-tier cellular networks. Fig. 2 also shows that for both the proposed interference minimized user association scheme and the conventional scheme, with more severe path loss effect, the interference caused to other co-channel MSs is reduced.

In Fig. 3, we show how interference is affected by various values of N_{T_y}. A greater N_{T_y} means that there are more candidate BSs to chose from such that it is more probably to select a BS which leads to less interference to other co-channel receiving MSs. Whereas when $N_{T_y} \to 1$, the proposed scheme degenerates to the conventional scheme.

6 Conclusions

In this paper, we first propose a transmission power normalization analysis model, which significantly simplifies the analysis of the received signal and interference, thus SIR, in multi-tier HCNs. Then we propose an interference minimized user association scheme, which can be applied in both single-tier and multi-tier HCNs. Using the proposed TPNM, we proceed to analyze the interference in multi-tier HCNs. Results demonstrate that the proposed scheme significantly reduces the down-link interference in multi-tier HCNs, meanwhile the constraint on the QoS of users is satisfied.

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