Heterogeneous On-Off sources in the Bufferless Fluid Flow Model

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Abstract

There are several methods to analyze the cell loss in a network: queuing analysis, large deviation approximation and bufferless fluid flow approximation. Bufferless fluid flow approximation is frequently used in literature for its simplicity. However, there is no effective means to investigate the cell loss in the bufferless fluid flow model. In this paper, we define the cell loss rate function (clrf) and suggest it for studying the cell loss of traffic sources in the bufferless fluid flow model. Properties of clrf are discussed. These properties enable us to decompose the complex analysis of the aggregation of several traffic sources into the simpler analysis of the individual sources. An important theorem about heterogeneous On-Off sources is proved using the clrf. The applications of the clrf and the theorem in cell loss analysis and connection admission control are presented. A real-time CAC scheme using on-line measurements is designed. This CAC scheme needs only simple information about traffic sources. Simulation studies are presented which are indicative of good performance of the CAC scheme.

1 Introduction

The Asynchronous Transfer Mode (ATM) is widely acknowledged as the base technology for the next generation of global communications. Since ATM adopts the statistical multiplexing technique, traffic control is necessary to avoid possible congestion at each network node and to achieve the quality-of-service (QoS) requested by each connection. Connection admission control (CAC) is a preventive traffic control mechanism and determines whether the network should accept a new connection or not. A new connection will only be accepted if the network has sufficient resources to meet its QoS requirements without affecting the QoS commitment already made by the network for existing connections.

Cell loss and cell delay are often adopted as a measure of QoS. Cell delay can usually be controlled within a desired bound by engineering the buffer size, hence in this paper we choose cell loss as the QoS[1], [2],[3], [4].

There are several methods to study cell loss: queuing analysis, large deviation approximation and bufferless fluid flow approximation. Bufferless fluid flow approximation is used in many CAC schemes for its simplicity[3], [4]. However there is no effective and efficient means to study the cell loss in the bufferless fluid flow model. In this paper, we propose the cell loss rate function (clrf) for estimating cell loss in the bufferless fluid flow model. The proposed clrf has many attractive features which greatly facilitates cell loss analysis and simplifies CAC algorithm. An important conjecture about heterogeneous On-Off sources in the bufferless fluid flow model proposed by Rasmussen et al. [5] is proved using the clrf. The applications of the theorem and the clrf in cell loss analysis are presented. Furthermore we propose a CAC scheme using on-line measurements. The calculations involved in the CAC scheme is very simple which makes it suitable for practical implementation in ATM network

The rest of the paper is organized as follows: in section two we propose the cell loss rate function, its properties are discussed; the clrf is applied to prove the conjecture in section three; the applications of the clrf and the theorem in cell loss analysis are presented in section four; in section five we propose the CAC scheme and a simulation study is presented; finally, some conclusions are drawn in section six.

2 Bufferless Fluid Flow Model

Under the bufferless fluid flow model, cell loss due to overflow occurs if and only if the sum of the traffic rates of all active connections denoted by R exceeds the link capacity C. Define the function $F(m)$ as follows:

$$\begin{aligned} F(m) &\triangleq E\left[(R-m)^+\right] \\ &\triangleq \sum_x (x-m)^+ f(x) \end{aligned} \qquad (1)$$

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where $f(x)$ denotes the traffic density distribution of a traffic and $(\bullet)^+$ denotes $max\{0, \bullet\}$. We call $F(m)$ the cell loss rate function (clrf) of $f(x)$. The clrf has many attractive features which facilitates our analysis of cell loss in the bufferless fluid flow model. For example, $F(C)$ denotes the cell loss rate of a traffic with traffic density distribution $f(x)$. Traffic sources with similar clrf can be regarded as equivalent in the cell loss analysis. From the definition of clrf we can easily derive that:

$$F(m) = F(0) - m; \quad \text{for} \quad m < 0 \tag{2}$$

$$F(0) = \rho = \sum_i mcr_i \tag{3}$$

where ρ denotes the traffic load and mcr_i denotes the mean cell rate of connection i on the link. The cell loss ratio (clr) can then be calculated as:

$$clr = \frac{F(C)}{\rho} = \frac{F(C)}{F(0)} \tag{4}$$

We can now derive several properties of clrf.

Property 1: Let $f(x)$ and $g(x)$ denote the traffic density distribution of independent traffic sources X_1 and X_2 respectively. Then, the cell loss rate function of $f(x) * g(x)$ equals $F(x) * g(x)$.

Proof: Construct a function $h(x)$ such that:

$$h(x) = \begin{cases} -x \quad ; & x < 0 \\ 0 \quad ; & x \geqslant 0 \end{cases} \tag{5}$$

It can be shown that:

$$F(m) = h(m) * f(m) \tag{6}$$

Thus we have the cell loss rate function $FG(m)$ of $f(x) * g(x)$:

$$\begin{aligned} FG(m) &= \sum_x (x-m)^+ (f(x) * g(x)) \\ &= h(m) * (f(m) * g(m)) \\ &= (h(m) * f(m)) * g(m) \\ &= F(m) * g(m) \end{aligned} \tag{7}$$

■

Property 2: Let $f(x)$, $g(x)$ and $q(x)$ denote the traffic density distribution of independent traffic sources X_1, X_2 and X_3 respectively. Denote the cell loss rate function of $f(x)$, $g(x)$, $f(x) * q(x)$ and $g(x) * q(x)$ by $F(m)$, $G(m)$, $FQ(m)$ and $GQ(m)$ respectively. If for $F(m)$ and $G(m)$ the condition $F(m) \geqslant G(m)$ holds for any m, then $FQ(m) \geqslant GQ(m)$ for any m.

Proof: Note that clrf is always non-negative. Thus proof of this property is straightforward. ■

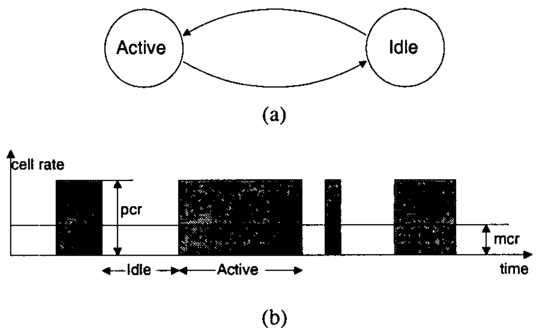

Figure 1. On-Off Source (a) source model (b) traffic model.

The properties of the clrf enable us to decompose the complex analysis of the aggregation of several traffic sources into the analysis of individual traffic source, hence the analysis is greatly simplified. The properties of clrf also greatly simplify the calculations involved in CAC algorithm.

3 Heterogeneous Bernoulli Sources in the Bufferless Fluid Flow Model

For simplicity let us consider On-Off sources. On-Off traffic source model is extensively used in CAC schemes [3] - [5] for its simplicity, and it has been successfully used to characterize the ON/OFF nature of an individual source or source element, like packetized voice and video [6], [7]. An On-Off source generates cells at peak cell rate denoted by *pcr* during active period. During idie period no cells are generated. Let *mcr* denote the mean cell rate of the On-Off source. We define the activity parameter of the On-Off source as:

$$p = \frac{mcr}{pcr}. \tag{8}$$

Then the probability that the On-Off source is active or idle is given by p or $1-p$ respectively. The pdf of the On-Off source is given by:

$$f(x) = \begin{cases} p \quad ; & x = pcr \\ 1-p \quad ; & x = 0 \\ 0 \quad ; & otherwise \end{cases}. \tag{9}$$

Fig. 1 illustrates the source model and traffic model of an On-Off source.

Theorem 1: Let $X_1, \cdots, X_N$ be N independent Bernoulli sources[1] with activity parameters $p_1, \cdots p_N$ respectively.

[1]By Bernoulli sources we mean On-Off sources with the same peak cell rate.

Their activity parameters are subject to $p_1 + \cdots + p_N = P$. Then in the bufferless fluid flow model, maximum cell loss will occur when $p_1 = \cdots = p_N = P/N$.[2]

Proof: The theorem is proved if we can prove that the clrf of N homogeneous Bernoulli sources, with activity parameter $p = P/N$, is greater than or equal to the clrf of the heterogeneous sources in the theorem for any m and N.

Step 1: First let us consider the case where $N = 2$. We need to prove that the clrf of two heterogeneous Bernoulli sources with activity parameters p_1 and p_2 and the same peak cell rate *pcr* is less than the clrf of two homogeneous Bernoulli sources with activity parameter $p = (p_1 + p_2)/2$ and peak cell rate *pcr*.

Their traffic density distribution functions are:

$$f_2^{(p_1,p_2)}(x) = \begin{cases} p_1 p_2 & ; x = 2pcr \\ (1-p_1)p_2 + (1-p_2)p_1 & ; x = pcr \\ (1-p_1)(1-p_2) & ; x = 0 \\ 0 & ; else \end{cases}$$

$$f_2^{(p)}(x) = \begin{cases} p^2 & ; \quad x = 2pcr \\ 2p(1-p) & ; \quad x = pcr \\ (1-p)^2 & ; \quad x = 0 \\ 0 & ; \quad else \end{cases}$$

where we use the superscript $(p_1, \cdots, p_n)$ to denote the activity parameters of the Bernoulli sources, and subscript n to denote the number of Bernoulli sources. We have the following cases to consider:

- when $m < 0$,

$$F_2^{(p_1,p_2)}(m) = F_2^{(p)}(m) = pcr \times (p_1 + p_2) - m$$

- when $0 \leqslant m < pcr$,

$$\begin{aligned} F_2^{(p_1,p_2)}(m) &= pcr \times (p_1 + p_2) \\ &- m(p_1 + p_2 - 3p_1 p_2) \\ F_2^{(p)}(m) &= 2pcr \times p - m(2p - 3p^2) \end{aligned}$$

$$F_2^{(p)}(m) - F_2^{(p_1,p_2)}(m) = 3m[\frac{(p_1+p_2)^2}{4} - p_1 p_2] \geqslant 0$$

- when $pcr \leqslant m < 2pcr$,

$$\begin{aligned} F_2^{(p_1,p_2)}(m) &= (2pcr - m)p_1 p_2 \\ F_2^{(p)}(m) &= (2pcr - m)p^2 \end{aligned}$$

$$\begin{aligned} &F_2^{(p)}(m) - F_2^{(p_1,p_2)}(m) \\ &= (2pcr - m)[\frac{(p_1+p_2)^2}{4} - p_1 p_2] \geqslant 0 \end{aligned}$$

[2] This theorem was first proposed as a conjecture by Rasmussen et al. [5] and numerically validated in many literatures including [3].

- when $2pcr \leqslant m$,

$$F_2^{(p)}(m) = F_2^{(p_1,p_2)}(m) = 0$$

From the above discussion we conclude that

$$F_2^{(p)}(m) \geqslant F_2^{(p_1,p_2)}(m) \quad \text{for any } m,$$

Step 2: Suppose $F_N^{(p)}(m) \geqslant F_N^{(p_1,\cdots,p_N)}(m)$ holds for the case when $N = n$. Namely,

$$F_n^{(p)}(m) \geqslant F_n^{(p_1,\cdots,p_n)}(m); \quad \text{for any } m$$

where $p = (p_1 + \cdots + p_n)/n$.

Let us consider the case when $N = n + 1$. We have:

$$\begin{aligned} &F_{n+1}^{(p_1,\cdots,p_{n+1})}(m) \\ &= F_n^{(p_1,\cdots,p_n)}(m) * f_1^{(p_{n+1})}(m) \\ &\leqslant F_n^{(\frac{p_1+\cdots+p_n}{n})}(m) * f_1^{(p_{n+1})}(m) \\ &= \left\{\sum_x (x-m)^+ [f_{n-1}^{(\frac{p_1+\cdots+p_n}{n})}(x) * f_1^{(\frac{p_1+\cdots+p_n}{n})}(x)]\right\} \\ &\quad * f_1^{(p_{n+1})}(m) \\ &= \left\{\sum_x (x-m)^+ [f_{n-1}^{(\frac{p_1+\cdots+p_n}{n})}(x) * f_1^{(p_{n+1})}(x)]\right\} \\ &\quad * f_1^{(\frac{p_1+\cdots+p_n}{n})}(m) \\ &\leqslant F_n^{(\frac{(n-1)\frac{p_1+\cdots+p_n}{n} + p_{n+1}}{n})}(m) * f_1^{(\frac{p_1+\cdots+p_n}{n})}(m) \\ &\quad \cdots\cdots\cdots\cdots\cdots\cdots \end{aligned}$$

Define a sequence a_k so that the above procedure can be expressed as:

$$\begin{aligned} &F_{n+1}^{(p_1,\cdots,p_{n+1})}(m) \\ &\leqslant F_n^{(a_1)}(m) * f_1^{(a_0)}(m) \\ &\quad \cdots\cdots\cdots\cdots\cdots\cdots \\ &\leqslant F_n^{(a_k)}(m) * f_1^{(a_{k-1})}(m) \\ &\quad \cdots\cdots\cdots\cdots\cdots\cdots \end{aligned}$$

We can derive that:

$$a_k = \frac{(n-1)a_{k-1} + a_{k-2}}{n}$$

$$a_0 = p_{n+1} \quad \text{and} \quad a_1 = \frac{p_1+\cdots+p_n}{n}$$

Solving for a_k, when $k \geqslant 2$, we get:

$$\begin{aligned} a_k &= a_1 - \frac{1 - \left(-\frac{1}{n}\right)^{k-1}}{n+1}(a_1 - a_0) \\ &= \frac{p_1 + \cdots + p_n}{n} \\ &\quad - \frac{1 - \left(-\frac{1}{n}\right)^{k-1}}{n+1}\left(\frac{p_1 + \cdots + p_n}{n} - p_{n+1}\right) \end{aligned}$$

Thus we have:

$$\lim_{k\to\infty} a_k = \lim_{k\to\infty} a_{k-1} = \frac{p_1+\cdots+p_{n+1}}{n+1}$$

This means that when the above process goes on and on, the activity parameters of the n homogeneous Bernoulli sources and the single Bernoulli source in the above equations will converge to $\frac{p_1+\cdots+p_{n+1}}{n+1}$. So we conclude that:

$$\begin{aligned} & F_{n+1}^{(p_1,\cdots,p_{n+1})}(m) \\ \leqslant\ & F_n^{(\frac{p_1+\cdots+p_{n+1}}{n+1})}(m) * f_1^{(\frac{p_1+\cdots+p_{n+1}}{n+1})}(m) \\ =\ & F_{n+1}^{(\frac{p_1+\cdots+p_{n+1}}{n+1})}(m) \end{aligned}$$

Hence from the supposition that $F_N^{(p)}(m) \geqslant F_N^{(p_1,\cdots,p_N)}(m)$ holds for the case when $N = n$, we derive the conclusion that the inequality holds for the case when $N = n+1$.

Combining step 1 and step 2 we conclude that:

$$F_N^{(p)}(m) \geqslant F_N^{(p_1,\cdots,p_N)}(m) \quad \text{for all } m \text{ and } n$$

Thus the theorem is proved. ■

The significance of the theorem is pointing out that the cell loss of heterogeneous Bernoulli sources is upper bounded by their corresponding homogeneous Bernoulli sources. In real network, many traffic sources of the same type have the same peak cell rate, however because of their specific application environments, they have different mean cell rates. Theorem 1 is very useful for analyzing this kind of traffic sources.

4 Applications in Cell Loss Analysis

Consider two independent On-Off sources X_1 and X_2 with peak cell rates pcr_1 and pcr_2 respectively, which have the same mean cell rate. Let X_3 be another independent traffic source. X_3 is not necessarily an On-Off source and may be the composition of several On-Off sources. It is shown [4] that if $pcr_1 \geqslant pcr_2$ then the cell loss due to the multiplexing of X_1 and X_3 is not less than that resulting from the multiplexing of X_2 and X_3. This conclusion can also be easily proved using our clrf.

Then let us consider N independent heterogeneous On-Off sources with peak cell rates $pcr_1, \cdots, pcr_N$ and mean cell rates $mcr_1, \cdots, mcr_N$ respectively. Let $pcr = max\{pcr_1, pcr_2, \cdots, pcr_N\}$, $mcr = (mcr_1 + mcr_2 + \cdots + mcr_N)/N$. It can easily be derived that the cell losses due to the multiplexing of these N heterogeneous On-Off sources are upper bounded by those due to the multiplexing of N independent heterogeneous On-Off sources with the same peak cell rate pcr and mean cell rates $mcr_1, \cdots, mcr_N$, which in turn are upper bounded by the cell losses due to the multiplexing of N independent homogeneous On-Off sources with peak cell rate pcr and mean cell rate mcr. Hence we arrive at the following theorem:

Theorem 2: Let $X_1, \cdots, X_N$ be N independent heterogeneous On-Off sources. Let pcr be the maximum of their peak cell rates and S be the sum of their mean cell rates. Then the cell losses due to the multiplexing of $X_1, \cdots, X_N$ is less than or equal to those due to the multiplexing of N independent homogeneous On-Off sources with peak cell rate pcr and mean cell rate S/N.

This theorem gives a cell loss upper bound for heterogeneous On-Off sources and can be used to design a very simple CAC algorithm.

5 Connection Admission Control Algorithm

Select a traffic rate unit u which equals the maximum peak cell rate of all calls in the network. From now on in this paper, all traffic rates are normalized with respect to u. Suppose there are N connections currently in the network, and denote the sum of their mean cell rates as S. Then their cell losses are upper bounded by N homogeneous On-Off sources with activity parameter $p = \frac{S}{N}$ and peak cell rate 1. The traffic distribution of the N homogeneous On-Off sources is the binomial distribution $f(k) = \binom{N}{k} p^k(1-p)^{N-k}$. Denote the clrf of $f(k)$ as $F(m)$ which can be calculated as follows:

$$\begin{aligned} F(0) &= S \\ F(m) &= F(0) - m & \text{for} \quad m < 0 \\ F(m) &= F(m-1) - 1 + \textstyle\sum_{j=0}^{m-1} f(k) & \text{for} \quad m > 0 \end{aligned} \tag{10}$$

Then the cell loss ratio of the N homogeneous On-Off sources in a link with bandwidth C is given by[3]:

$$clr = \frac{F(C)}{F(0)}. \tag{11}$$

The cell loss ratio of the N connections in the network is upper bounded by clr.

The sum of the mean cell rates S required for the CAC scheme can be inferred from either the traffic descriptors of connections in the network at connection setup phase or from on-line measurements. Considering that in real networks it is very difficult for traffic sources to characterize their mean cell rates accurately we derive S from on-line measurements in our CAC scheme. S is the mean traffic rate of the network which can be easily measured.

[3] Without loss of generality, here we assume that C is an integer.

When a new connection with mean cell rate mcr_{new} arrives, the cell loss ratio after its admission is estimated as:

$$\widehat{clr} = \frac{\widehat{F}(C)}{\widehat{F}(0)} \qquad (12)$$

where

$$\widehat{F}(C) = (1 - mcr_{new})\, F(C) + (mcr_{new})\, F(C-1)$$
$$\widehat{F}(0) = F(0) + mcr_{new}$$

If the estimated cell loss ratio is less than the cell loss ratio objective, the new connection is admitted, otherwise the new connection will be rejected.

If the new connection is admitted, clrf will be updated:

$$\begin{aligned} \widehat{F}(0) &= F(0) + mcr_{new} \\ \widehat{F}(m) &= \widehat{F}(0) - m \qquad \text{for } m < 0 \\ \widehat{F}(m) &= (1 - mcr_{new})\, F(m) \\ &+ mcr_{new} F(m-1) \qquad \text{for } m > 0 \end{aligned} \qquad (13)$$

5.1 Simulation Study

CAC scheme using on-line measurements can only be verified through experiments on either real networks or simulation. In this section, we study the performance of the CAC scheme using simulation. The aim of the simulation study is to evaluate the performance of the CAC scheme with respect to network utilization and the effectiveness of the CAC scheme in terms of its ability to guarantee the QoS constrains required by the connections.

Exponential On-Off sources are used for our simulation. The duration of the ON and OFF periods are independent and exponentially distributed with means β and γ respectively. During each ON period an exponentially distributed random number of cells, with mean L, are generated at peak cell rate. During off periods no cells are generated. We define the burstiness of a traffic source as:

$$Burstiness = \frac{pcr}{mcr} = \frac{\beta + \gamma}{\beta} \qquad (14)$$

Furthermore, the following parameters are used for our simulation: cell loss ratio objective is set to 10^{-4}, the link capacity is set to $10Mb/s$, and the measurement interval is 0.05 second. Switching speed of the ATM switch is set to infinity, hence every incoming cell is placed immediately in the output buffer. The output buffer size is set to 20 cells to absorb cell level congestion[8]. The link utilization and cell loss ratio are observed in our simulation study.

In our simulation, three types of traffic sources are multiplexed onto the link. Each type of traffic has an exponentially distributed arrival rate with mean λ calls per second. The connection holding time for all traffic types is exponentially distributed with a mean of 100 seconds. The call arrival rate is set to be very high. The high call arrival rate and long call holding time mean that the system is continually receiving new connection requests. Thus we expect the link to remain at close to maximum utilization. We also want to establish the QoS performance of our CAC scheme by choosing the high call arrival rate, since the CAC schemes are expected to perform worse with respect to QoS under high call arrival rates. The parameters of the three traffic types are listed in Table 1.

Table 1. Parameters of the three traffic types

	$\lambda(s^{-1})$	$pcr(kb/s)$	$burstiness$	$L(cells)$
type 1	10	100	10	100
type 2	50	50	5	50
type 3	100	10	2	20

Note that the mean burst length is several times larger than the buffer size. This is used as a trial to establish the performance of the CAC scheme using on-line measurements.

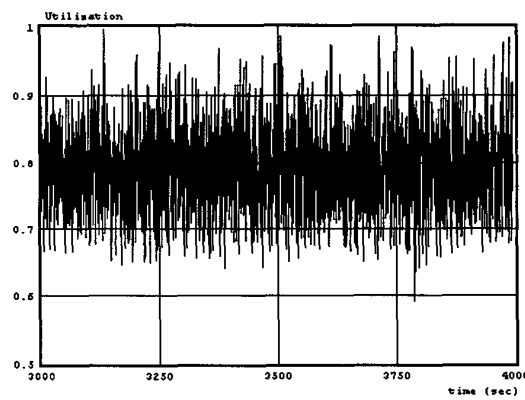

Figure 2. Utilization

The simulation was run for 10,000 seconds. Fig. 2 shows the observed utilization during the period 3,000 to 4,000 seconds. We define the statistical multiplexing gain as the ratio of the utilization achieved by our CAC scheme to that can be achieved by peak cell rate CAC algorithm under the same environment. An average statistical multiplexing gain of 3.4 is achieved by our CAC scheme. Fig. 3 shows the observed cell loss ratio. Fig. 4 shows the number of each connection type on the link. The number of connections varies within a large range. As shown in Fig. 3, the proposed CAC scheme always maintain tight QoS commitment to all connections which indicates that the CAC scheme using on-line measurements is robust. Simulation also shows that the CAC scheme is time efficient and suits real-time requirement of ATM networks.

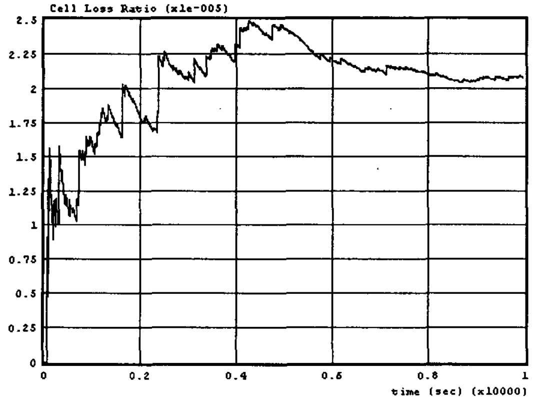

Figure 3. Cell Loss Ratio

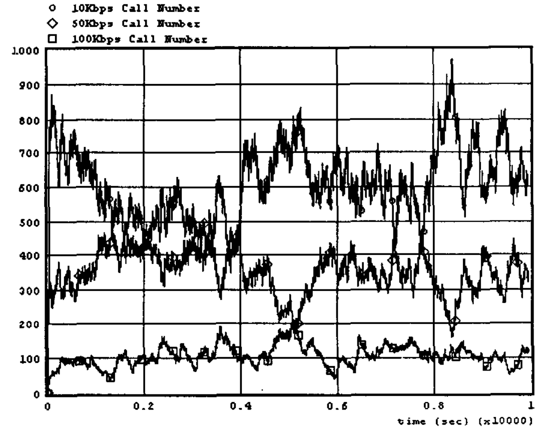

Figure 4. The Number of the three type connections in the link

Thus we have shown that the proposed CAC scheme using on-line measurements is robust, efficient and capable of achieving a high network utilization.

6 Conclusion

In this paper, we proposed the cell loss rate function as a tool for studying the cell loss in the bufferless fluid flow model. Properties of the cell loss rate function were discussed. These properties enable us to decompose the complex analysis of the aggregation of several traffic sources into the simpler analysis of the individual sources. Moreover, the clrf can greatly simplify the calculations involved in the CAC scheme. A theorem for heterogeneous Bernoulli sources was proved using the clrf. The theorem is very useful for analyzing cell loss in real networks.

Furthermore, the applications of the clrf and the theorem in cell loss analysis and CAC schemes were presented. The theorem was extended to heterogeneous On-Off sources. A very simple CAC scheme using on-line measurements was proposed on the basis of these theoretical analysis. Simulation studies showed that the proposed CAC scheme is both robust and time efficient.

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