

Analysis of Access and Connectivity Probabilities in Infrastructure-Based Vehicular Relay Networks

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Abstract—Coverage is an important problem in wireless networks. Together with the access probability, which measures how well an arbitrary user can access a wireless network, in particular VANET, they are often used as major indicators of the quality of the network. In this paper, we investigate the coverage and access probability of the vehicular networks with roadside infrastructure, i.e. base stations. Specifically, we analyze the relation between these key parameters, i.e. the coverage range of base stations, coverage range of vehicles, vehicle density and distance between adjacent base stations, and how these parameters interact with each other to collectively determine the coverage and the access probability. We use the connectivity probability, the probability that all nodes in the network are connected to at least one base station within a designated number of hops, as a measure of the coverage. We derived close-form formulas for the connectivity probability and the access probability for a 1D vehicular network bounded by two adjacent base stations. The analytical results have been validated by simulations. The results in the paper can be used by network operators to design networks with specific service coverage guarantees.

Index Terms—Vehicular Ad Hoc Network (VANET), Wireless Access in Vehicular Environments (WAVE), IEEE 802.11p, access probability, connectivity, relay.

I. INTRODUCTION

Vehicular ad-hoc network (VANET) is a promising application-oriented network deployed along a highway for safety and emergency information delivery (for drivers), entertainment content distribution (for passengers), and data collection and communication (for road and traffic managers). VANET is a kind of hybrid wireless network that supports both infrastructure-based and ad-hoc network structures and communication methods. Specifically, vehicles on the road can communicate with each other through a multi-hop ad-hoc connection (path). They can also access the Internet and other broadband services through the roadside infrastructure,

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i.e. base stations (BSs) or access points (APs) along the road. When a vehicle is outside the radio coverage area of BSs, i.e. it is located in the coverage gap between adjacent BSs, it will identify and use its neighboring vehicles (if any) as relay to access the roadside infrastructure.

IEEE 802.11p is a new standard currently under development in order to provide wireless access in vehicular environments (WAVE) and to deliver safety and infotainment applications to the vehicles on the road [1]. According to the US Department of Transportation, “the WAVE (802.11p) standards define an architecture and a complementary, standardized set of services and interfaces of this kind of wireless access that collectively enable vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) wireless communications” [2].

Variable vehicle speed, high mobility, and dynamically-changing network topology are the main characteristics of VANETs. They are therefore the key technical challenges in developing and deploying WAVE applications and services. From a user/vehicle’s perspective, the first, and probably the most important service requirement is to be able to access the roadside infrastructure (i.e. BSs) directly or indirectly via relay vehicles. From the perspective of a network operator or service provider, it is very important to guarantee satisfactory and profitable service coverage while minimizing the deployment and maintenance costs of the roadside infrastructure, which are mainly determined by the density and complexity of BSs.

In [3], the authors find that a one-hop link can achieve better spectral efficiency than a multi-hop communication connection, when the signal-to-noise ratio (SNR) is relatively high. The outage probability and spectral efficiency performance over one-hop and two-hop connections are further analyzed in a simple wireless network with fading [4]. In [5], the author evaluates the IEEE 802.11p WAVE standard in terms of collision probability, throughput and delay performance. Network and service connectivity probability has been analyzed for wireless sensor networks [6] and VANET [7]. In [7], an ad hoc V2V communication scenario is considered in the system modeling and performance analysis, while our focus in this paper is to study the performance of V2I communications with relays.

Specifically, this research aims to address those challenging problems in an infrastructure-based vehicular relay network, wherein only one-hop (direct access) and two-hop (via a relay)

communications between a vehicle and a BS are supported in order to reduce network complexity and improve quality of service (QoS) in vehicular environments. We first develop an analytical model of such a vehicular network and derive user access probability and service connectivity probability. Further, we study the impact of inter-BS distance (or BS density), the radio coverage ranges of a BS and a vehicle, and the density of the vehicles on these important performance metrics.

The rest of this paper is organized as follows. Section II defines the system model of infrastructure-based vehicular relay networks. Access and connectivity probabilities are analyzed under four communication scenarios in Section III. Section IV presents and discusses our analytical and simulation results, followed by conclusions in Section V.

II. SYSTEM MODEL

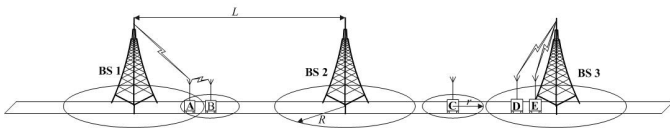


Fig. 1. An Infrastructure-Based Two-hop Vehicular Network.

We consider an infrastructure-based vehicular relay network, as shown in Fig. 1, wherein a number of BSs are uniformly deployed along a long road, while other vehicles or cars are distributed on the road according to a Poisson distribution [6]. We analyze the access probability and the connectivity probability of the network by investigating a subnetwork bounded by two adjacent base stations. Let L be the Euclidean distance (in meters) between two adjacent BSs and let ρ be the vehicle density in terms of vehicles per meter (vpm). So the probability that k vehicles are found in a distance of x meters is given as

$$f(k, x) = \frac{(\rho x)^k e^{-\rho x}}{k!}, k \geq 0. \quad (1)$$

As a special case of the above formula, the probability that an interval with length x in this road segment have no vehicles is

$$g(x) = f(0, x) = e^{-\rho x}. \quad (2)$$

In addition, the Euclidean distance between a reference point and the first occurrence of a vehicle (Poissonly distributed point) after the reference point is exponentially distributed. That is, the event that the vehicle, which is closest (in Euclidean distance) to a reference point, is x Euclidean distance away from the reference point has pdf (probability density function)

$$h(x) = \rho e^{-\rho x}. \quad (3)$$

Following a simple disk communication model, the radio coverage radii of a BS and a vehicle are fixed as R and r meters ($R \geq r$), respectively. When a vehicle is located within the radio coverage range of a BS, it will be able to directly

communicate with this BS, i.e. transmit to and receive from the BS, even though the distance between them may be larger than r . This assumption is justified that it is often the case that a BS can not only transmit at a larger transmission power, it can also be equipped with more sophisticated antennas, which make it more sensitive to the transmitted signal from a vehicle. Alternatively, the vehicle can slightly increase its transmission power to establish a one-hop direct link with the BS and, hence, achieve better QoS performance, comparing to a two-hop communication connection.

Referring to Fig. 1, three BSs and five vehicles (denoted by A, B, C, D, and E) are seen on a segment of the road. Vehicles A, D and E are located in the direct coverage ranges of their neighboring BSs, so they are called “direct vehicles” (DVs). Vehicle B is located in the coverage gap between BS1 and BS2, it uses a neighboring DV (i.e. vehicle A) as its relay to access BS1. Such vehicles with a relay are defined as “two-hop vehicles” (THVs) in the network. Finally, vehicle C in the coverage gap between BS2 and BS3 cannot find any DVs in its vicinity, so it cannot access either BS and is called an “isolated vehicle” (IV). Access probability is the probability that an arbitrary vehicle is not an IV. On the other hand, connectivity probability is the probability that none of the vehicles are IVs.

III. ACCESS AND CONNECTIVITY PROBABILITY

For a typical vehicle (say vehicle V) on the road, its access probability p_a to a BS is determined by the values of the radio coverage radii R and r , the inter-BS distance L , and the vehicle density ρ . These system parameters also determine the network/service connectivity probability, denoted by p_c , for the whole network. Without loss of generality, we consider a road segment between two adjacent BSs and four communication scenarios. For the ease of discussion, we assume the road segment being considered is $[0, L]$, and the BSs are labeled as BS1 and BS2 respectively.

A. $0 < L \leq 2R$

In this case, all the vehicles on the road can directly communicate with a BS, so the access probability of vehicle V is $p_a = 1$, and the connectivity probability is simply $p_c = 1$.

B. $2R < L \leq 2R + r$

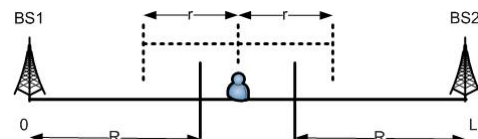


Fig. 2. A radio coverage gap between adjacent BSs.

In this case, as shown in Fig. 2, the radio coverage gap between two adjacent BSs is not more than the coverage radius of a vehicle. The probability that vehicle V is located in the coverage of either BS (i.e. V is a DV) is simply $2R/L$. Otherwise, vehicle V is located in the coverage gap, but its radio coverage has overlaps with both BSs. The probability

that vehicle V can find at least one DV in its vicinity as a relay is given by $1 - e^{-\rho(2R+2r-L)}$. So the access probability of vehicle V can be derived as

$$\begin{aligned} p_a &= \frac{2R}{L} + \frac{L-2R}{L}(1 - e^{-\rho(2R+2r-L)}) \\ &= 1 - \frac{L-2R}{L}e^{-\rho(2R+2r-L)} \end{aligned} \quad (4)$$

To obtain connectivity probability, we divide the event that all vehicles in the road segment can communicate with at least one of the two BSs into three mutually exclusive cases.

- 1) No vehicles exist in the road segment.
- 2) All vehicles can communicate with BS1 in at most two hops, but not BS2. And symmetrically, all vehicles can communicate with BS2 in at most two hops, but not BS1.
- 3) For each BS, there is at least one vehicle which can directly communicate with it. All vehicles can communicate with either BS1 or BS2 in at most two hops or both.

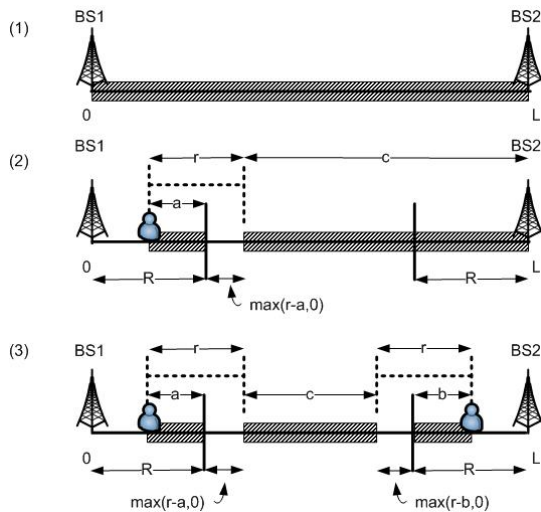


Fig. 3. The three possible cases. Note that the symmetric case for case 2 is not shown.

Fig. 3 shows the aforementioned three possible cases, illustrated by each subgraph. In general, the shaded intervals represent that no vehicles shall exist in those intervals. Fig. 3-1 has the whole interval between BS1 and BS2 shaded, to represent that there is no vehicle exists in the road segment between the BSs. Fig. 3-2 illustrates that there is a vehicle which is $R-a$ Euclidean distance away from BS1. This vehicle is assumed to be the vehicle furthest away from BS1 that can directly communicate with BS1. Hence, the interval of length a is shaded. In case 2, we further assume that vehicles which cannot communicate with any of the BSs or can communicate with BS2 does not exist, which results in the shading of the interval of length c . Fig. 3-3 assumes that the furthest one-hop vehicle to BS1 is $R-a$ Euclidean distance away from BS1, and the furthest one-hop vehicle to BS2 is $R-b$ Euclidean distance away from BS2, then there may exist an interval of length c

in which no vehicles shall exist. Otherwise the vehicles in that interval cannot communicate with any of the BSs in at most two hops. Such interval will not exist if the communication range of the one-hop vehicles can fully cover the interval which is not covered by the communication range of the BSs. That is exactly when $2R + \max(r-a, 0) + \max(r-b, 0) \geq L$.

We can derive the probabilities of occurrence for the three cases mentioned. The probability that case 1 happens is

$$p_1 = g(L). \quad (5)$$

The probability that case 2 happens is

$$p_2 = \int_0^R h(a)g(c)da \quad (6)$$

where $c = \max(L - R - \max(r-a, 0), R)$ and the feasible range of a is from 0 to R . The probability that case 3 happens is

$$p_3 = \int_0^R \int_0^R h(a)h(b)g(c)dbda \quad (7)$$

where $c = \max(L - 2R - \max(r-a, 0) - \max(r-b, 0), 0)$. So, the probability that all vehicles can reach at least one base station in at most two hops is then

$$p_c = p_1 + 2p_2 + p_3 \quad (8)$$

with inclusion of the symmetric case. With some arithmetic steps, we can obtain p_c for $2R < L \leq 2R + r$

$$\begin{aligned} p_c &= 1 + \frac{1}{4}e^{-\rho(L-2R+2r)} \\ &\quad - \left[\frac{1}{4} + \frac{\rho(L-2R)}{2} \right] e^{-\rho(2R+2r-L)} \end{aligned} \quad (9)$$

C. $2R + r < L \leq 2R + 2r$

In this case, when vehicle V is located in the radio coverage gap between BS1 and BS2, its coverage area may overlap with the coverage area of only one of the BSs (with a probability of $(2L-4R-2r)/L$), or the coverage areas of both BSs (with a probability of $(2R+2r-L)/L$). Depending on the exact location of vehicle V, the overall access probability is given by

$$\begin{aligned} p_a &= \frac{2R}{L} + \frac{2}{L} \int_{2R+2r-L}^r (1 - e^{-\rho x}) dx \\ &\quad + \frac{2R+2r-L}{L}(1 - e^{-\rho(2R+2r-L)}) \\ &= 1 + \frac{2}{\rho L}(e^{-\rho r} - e^{-\rho(2R+2r-L)}) \\ &\quad - \frac{2R+2r-L}{L}e^{-\rho(2R+2r-L)} \end{aligned} \quad (10)$$

Similar to the previous scenario, we can obtain the connectivity probability

$$\begin{aligned} p_c &= 1 + \frac{1}{4}e^{-\rho(L-2R+2r)} + \frac{1}{2}e^{-\rho(L-2R)} \\ &\quad - \left[\frac{3}{4} + \frac{\rho(2R+2r-L)}{2} \right] e^{-\rho(2R+2r-L)} \end{aligned} \quad (11)$$

D. $L > 2R + 2r$

In this case, with a probability of $(L - 2R - 2r)/L$, vehicle V is located in a region where its radio coverage has no overlap with that of either BS1 or BS2, i.e. vehicle V cannot reach any DVs and, therefore, cannot access to either BS. With a probability of $2r/L$, vehicle V has an overlapped coverage area with one BS. So, the overall access probability can be calculated as

$$p_a = \frac{2R}{L} + \frac{2}{L} \int_0^r (1 - e^{-\rho x}) dx = \frac{2R + 2r}{L} + \frac{2(e^{-\rho r} - 1)}{\rho L} \tag{12}$$

The connectivity probability is

$$p_c = \frac{1}{4}e^{-\rho(L-2R+2r)} + \frac{1}{2}e^{-\rho(L-2R)} + \frac{1}{4}e^{-\rho(L-2R-2r)} \tag{13}$$

IV. ANALYTICAL AND SIMULATION RESULTS

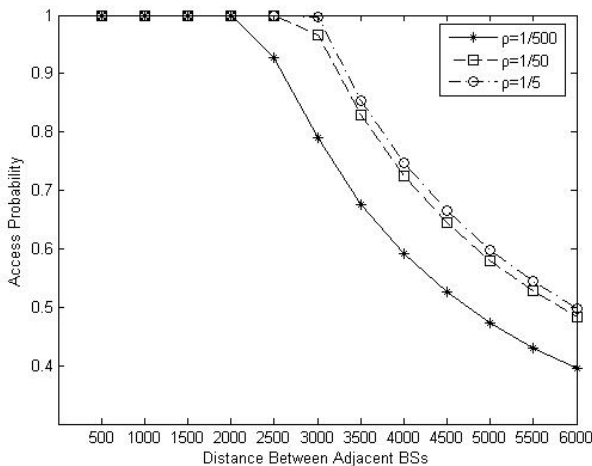


Fig. 4. Access Probability with L changing, $R = 1000$ meters, $r = 500$ meters, $\rho = 1/5, 1/50, 1/500$ cars/m respectively.

Fig. 4 shows the access probability given different values of L and ρ , plotted using the equations obtained in Section III. As shown in the figure, the access probability decreases with L when L exceeds some limits. For small ρ , the access probability decreases as soon as $L > 2R$. That is because when the vehicle density ρ (number of vehicles per meter) is low, a vehicle is either directly connected to a BS or disconnected, i.e cannot reach any BS in at most two hops. It is hard for the vehicle to find a one-hop relay in its coverage range via which it can access a BS if it is not within the coverage of any BS. However for large ρ , it is easier for the vehicle, which is not within the coverage of any BS, to find a one-hop relay to access the BS. In general ρ plays as a positive factor to the access probability, and the reason is that when the vehicle density increases, the probability increases for vehicles in the gap of the coverage areas of BSs to find a neighbor within the coverage area of a BS to act as a relay.

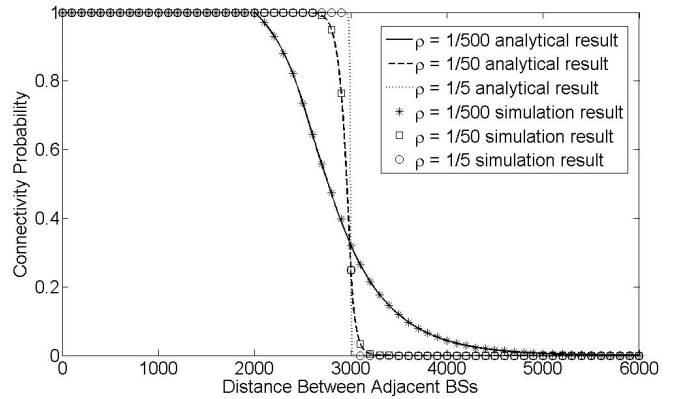


Fig. 5. Connectivity Probability with L changing, $R = 1000$ meters, $r = 500$ meters, $\rho = 1/5, 1/50, 1/500$ cars/m respectively.

Similarly, Fig. 5 shows the connectivity probability, the probability that all vehicles in the road segment of length L (in meters) can communicate with at least one base station in at most two hops, given different values of L and ρ . The analytical results are verified by the simulation results which are obtained from 40000 randomly generated network topologies. As the number of instances of random networks used in the simulation is very large, the confidence interval is too small to be distinguishable and hence ignored in this plot. The figure shows that when $L \leq 2R + r = 2500$ meters, it is easy for all vehicles to be connected to at least one base station in at most two hops, hence the connectivity probability is high. As L gets larger, it is harder for all vehicles to be connected to the base stations due to the larger possible distances between the vehicles and the base stations. This implies a drop in the connectivity probability, and the connectivity probability tends to zero as L goes to infinity. The transition of the connectivity probability from 1 to 0 as L gets sharper as the vehicle density increases. As ρ goes to infinity, the transition happens at the critical distance $L = 2R + 2r = 3000$, where below which the network is almost disconnected and above which the network is almost connected. Furthermore, the networks with a larger ρ have higher connectivity probability than the networks with a smaller ρ when L is small. This is because when the vehicle density is large, it is easy for vehicles not directly connected to a base station to find a vehicle within its communication range and is directly connected to a BS to act as a relay. When L is large, the networks with a larger ρ have a lower connectivity probability than the networks with a smaller ρ . This is because at large values of L when the vehicle density is large it is easy to have at least one vehicle which is located too far from the base stations to be connected to a BS in at most two hops.

Fig. 6 shows how the coverage radius of the vehicles r impacts on the access probability. It shows that the access probability increases with r , and when ρ is big enough, the access probability could be quite close to 1. And it shows again that ρ is a positive factor to the access probability.

With the similar setup, Fig. 7 shows the sensitivity of the

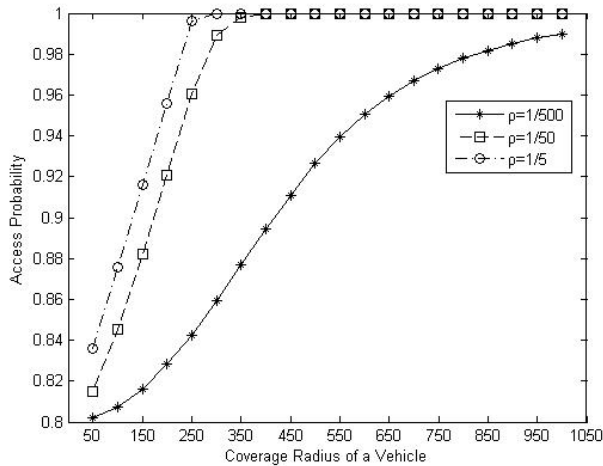


Fig. 6. Access Probability with r changing, $L = 2500$ meters, $R = 1000$ meters, $\rho = 1/5, 1/50, 1/500$ cars/m respectively.

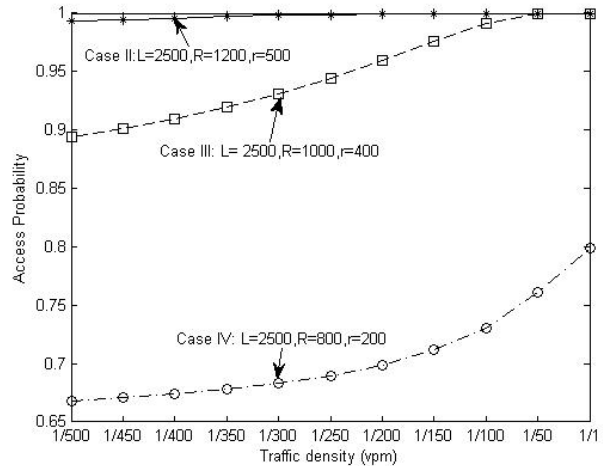


Fig. 8. Access Probability with ρ changing, L, R, r be different constant numbers.

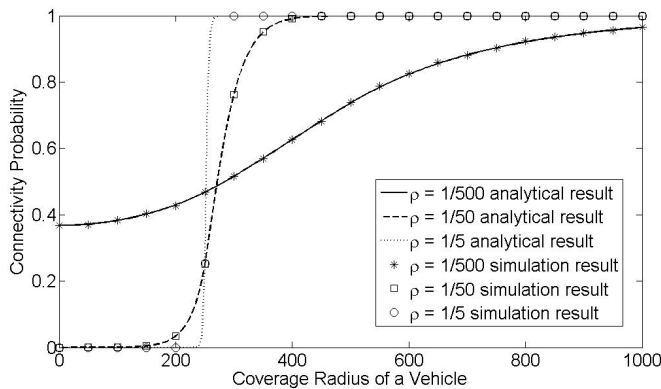


Fig. 7. Connectivity Probability with r changing, $L = 2500$ meters, $R = 1000$ meters, $\rho = 1/5, 1/50, 1/500$ cars/m respectively.

connectivity probability to r . For large ρ , around a certain value of r a small increment in r will incur a dramatic increase in the connectivity probability from near 0 to near 1. From the figure it shows that such phenomenon does not exist for small ρ .

Fig. 8 supported our conclusion that an increase in ρ will improve the access probability as it shows that the access probability monotonically increases with ρ . While ρ is relatively small, and the width of the gap region not directly covered by any of the base stations is relatively large, the access probability will be low, and thus, in this circumstance, network operator should consider to deploy more BSs along the highway for better coverage and greater access probability.

Theoretically by derivation of p_a , it is easy to observe that p_a monotonically increases with R, r and ρ , while it monotonically decreases with L when $L > 2R$.

So during the realistic deployment of the roadside network, it could be easy to decide how to choose the locations for BSs using our analytic results. Given that R, r are known (and it is usually practical), then we only need to find out ρ by field

test on the road to get an average vehicle density. And if the access probability should be no smaller than γ , then we can solve the inequality $p_a \geq \gamma$ to get L .

After the deployment L becomes a known constant, if a vehicle wants to increase its access probability, it should expand its coverage range by increasing its transmission power. Here we didn't consider the collisions between two simultaneous transmissions. BSs could also guarantee the access probability by increasing their transmission power using the equations derived in this paper; when the vehicle density is large, BSs can decrease their transmit power for power saving consideration while maintaining the same level of access probability and connectivity probability.

V. CONCLUSIONS AND FUTURE WORK

In this paper, we analyzed the connectivity probability and the access probability for a given network bounded by two adjacent base stations, and vehicles in the network are Poissonly distributed with known density and each vehicle can communicate with a base station in at most two hops. Under the unit disk communication model, we derived closed-form formulas for the access probability and connectivity probability considering that the base stations and the vehicles have different coverage ranges. These formulas characterize the relation between these key parameters, i.e. the coverage ranges of the base stations and the vehicles, the distance between adjacent base stations, the vehicle density and their impact on the access and connectivity probabilities. These results can be useful for a network operator to design a network with a given level of access guarantee.

The analysis in this paper is built on the basis of the unit disk model, which has been widely used in this area to obtain some fundamental results but too simplistic to capture real channel characteristics. In the future, we shall extend the work to include some more realistic channel models, including the widely used log-normal model.

REFERENCES

- [1] "IEEE P802.11 - task group p," http://grouper.ieee.org/groups/802/11/Reports/tgp_update.htm.
- [2] "IEEE 1609 - family of standards for wireless access in vehicular environments (WAVE)," http://www.standards.its.dot.gov/fact_sheet.asp?f=80, 2006.
- [3] A. Florea and H. Yanikomeroglu, "On the optimal number of hops in infrastructure-based fixed relay networks," in *IEEE GLOBECOM*, 2005, pp. 3242–3247.
- [4] S. V. Maiya and T. E. Fuja, "One-hop vs. two-hop routing in simple networks with fading: An outage probability analysis addressing spectral efficiency," in *IEEE WCNC*, 2008, pp. 494–499.
- [5] S. Eichler, "Performance evaluation of the IEEE 802.11p WAVE communication standard," in *IEEE Vehicular Technology Conference*, 2007, pp. 2199–2203.
- [6] X. Ta, G. Mao, and B. D. O. Anderson, "On the probability of k-hop connection in wireless sensor networks," *IEEE Communications Letters*, vol. 11, no. 8, pp. 662–664, 2007.
- [7] J. Wu, "Connectivity analysis of a mobile vehicular ad hoc network with dynamic node population," in *IEEE GLOBECOM Workshops*, 2008, pp. 1–8.