

On the Connectivity Properties of Wireless Multi-hop Networks

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Abstract—Given a multi-hop network in which a total of n nodes are randomly and independently distributed in a unit square following a uniform distribution and each node has a uniform transmission range $r(n)$, and two distinct nodes can directly communicate with each other if and only if their Euclidean distance is at most $r(n)$, this paper investigates the characteristics of the minimum transmission range $r_c(n)$, at which the network is connected with a high probability. We show that for small values of n , $r_c^2(n)$ grows approximately linearly with $\frac{1}{n}$; and as n goes to infinity, $r_c^2(n)$ scales with $\frac{\log n}{n}$. Simulations are performed to verify the theoretical analysis. The results of this paper are very useful in the design and dimensioning of wireless sensor networks and wireless ad hoc networks.

I. INTRODUCTION

Connectivity is one of the fundamental properties in wireless multi-hop networks (e.g., wireless sensor/ad hoc network), and is also a prerequisite for providing many network functions [1]. A question naturally follows is: what is the minimum transmission range that ensures a given network to be connected with a high probability? Or equivalently, what is the minimum value of the average node degree that ensures a given network to be connected with a high probability? Here the average node degree means the average number of neighbors of each node. For a network in which a total of n nodes are distributed in a unit area following a uniform distribution and each node has a uniform transmission range $r(n)$, and two distinct nodes can directly communicate iff (if and only if) their Euclidean distance is at most $r(n)$, the average node degree is related to the transmission range by $n\pi r^2(n)$ ignoring the boundary effect. Because of this relationship between the transmission range and the average node degree, we use the two terms exchangeably in the rest of the paper when there is no confusion.

On one hand, if the transmission range is too small, some nodes may become isolated and the network becomes disconnected; on the other hand, if the transmission range is too large, it will cause interference and waste energy. Hence, it is essential to set just enough transmission range such that the

network is connected with a high probability while causing minimal interference.

There has been significant research on the problems of the minimum transmission range and the average node degree required for connectivity. The previous results can be separated into two categories:

- Non-asymptotic (i.e., finite n) results: Koskinen [2] consider the network by uniformly distributing n nodes in a square with n varying from 5 to 350 and adding edge between any two nodes iff their Euclidean distance is at most the transmission range. They showed through simulations that the squared inverse of the mean of the minimum transmission range required for an instance of the network to be connected grows approximately with n . Assume that $\frac{1}{r^2(n)} \approx C \cdot n$ according to [2], where C is some constant, then $n\pi r^2(n) \approx \frac{\pi}{C}$. Their results imply that the average node degree required for a random network to be connected is approximately a constant (i.e., $\frac{\pi}{C}$). Similarly, Ni *et al.* [3] showed through simulations that if the average node degree is set to be some constant between 6 and 10, the resulting network is connected with high probability. This constant is called “magic number” by the authors. The network is modeled as nodes located randomly on a square of size 100×100 according to a Poisson point process. The node density is varied from 0.002 to 0.03.
- Asymptotic (i.e., infinite n) results: Gupta *et al.* [4] proved that if the transmission range is set to $r(n) = \sqrt{\frac{\log n + c(n)}{\pi n}}$, the resulting network is asymptotically connected with probability one iff $c(n) \rightarrow \infty$ as $n \rightarrow \infty$, where the network is formed by uniformly placing n nodes in a unit-area disc. Philips *et al.* [5] proved that the average node degree must grow logarithmically with the area of the network to ensure network connectivity, where the nodes are located randomly on a square according to a Poisson point process with a constant node density. Xue *et al.* [6] proved that each node should be connected to $\Theta(\log n)$ nearest neighbors if the network is to be connected, where n nodes are randomly distributed in a unit square following a uniform distribution. The results in [4], [5], [6] imply that if the average node degree is fixed to be some constant, then the network will almost

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surely be disconnected when n is sufficiently large.

These two kinds of results seem to be incompatible with each other. In this paper, we try to reconcile the two apparently contradictory results. Specifically, we investigate the characteristics of the minimum transmission range $r_c(n)$ at which the network is connected with a high probability. We show that $r_c^2(n)$ grows approximately linearly with $\frac{1}{n}$ for small values of n (e.g., $n = 25 \sim 300$); when n is sufficiently large, $r_c^2(n)$ must grow like $\frac{\log n}{n}$, or equivalently, the average node degree must grow like $\log n$ since the average node degree is $n\pi r^2(n)$ when the boundary effect is ignored. Our results are very useful in the design and dimensioning, and may provide guidance for transmission power control and routing in wireless sensor/ad hoc networks.

The rest of this paper is organized as follows. Section II describes the network model and some basic concepts of graph theory used in the paper. In Section III, we investigate the relationship between the square of the minimum transmission range and the number of nodes n . Section IV presents the simulation results. Finally, Section V concludes this paper.

II. PRELIMINARIES

A. Network model

For many purposes, a wireless multi-hop network can be represented by an undirected graph $G(V, E)$ with a set of vertices V and a set of edges E . Each vertex of the set V uniquely represents a node in the network and each edge of the set E uniquely represents a wireless link in the network, and vice versa. In the past several years, a so-called *random geometric graph* has been widely used to represent a wireless multi-hop network [1], [4], [7], [8], [9]. Throughout this paper, our network is modeled by a random geometric graph $G(n, r(n))$. And we assume that $n \gg 1$ (e.g., $n = 25, 50$) and $\pi r^2(n) \ll 1$.

Definition 1 ([10], [11]): Given $n \in \mathbb{N}$ and $r \in [0, 1]$, a random geometric graph $G(n, r)$ is a graph in which n vertices are randomly and independently distributed in a unit square in \mathbb{R}^2 following a uniform distribution, and any two vertices u and v are directly connected iff $\|u - v\| \leq r$, where the norm $\|\cdot\|$ means the Euclidean norm.

Due to the scaling property of random geometric graphs, any realization $G(n, r(n))$ in a *unit* square is equivalent to another realization $G(n, \sqrt[3]{Ar(n)})$ placed in a square of area A [11]. Hence, throughout this paper, we focus on $G(n, r(n))$ distributed in a *unit* square in \mathbb{R}^2 .

B. Concepts from graph theory

In this subsection, we briefly introduce some basic concepts in graph theory, which will be used later.

The *degree* of a node u , denoted as $d(u)$, is the number of its neighbors directly connected to it [12]. A node of degree zero is called an *isolated node* (refer to Fig. 1-i). The minimum node degree of a graph G is defined as

$$d_{\min}(G) = \min_{u \in V(G)} \{d(u)\},$$

and the average node degree of a graph G is

$$d_{\text{avg}}(G) = \frac{1}{n} \sum_{u \in V(G)} d(u),$$

which represents the average number of neighbors of an arbitrary node. If the boundary effect is ignored, we have $d_{\text{avg}}(G) = n\pi r^2(n)$.

A graph is said to be connected (or 1-connected) iff for any pair of two nodes there exists at least one path connecting them [12] (refer to Fig. 1-ii). A graph is said to be k -connected iff for any pair of two nodes there exists at least k mutually independent paths connecting them [12], i.e., these paths do not share a common node except for the beginning and the end of the path (refer to Fig. 1-iii).

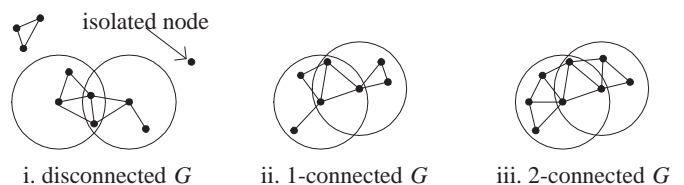


Fig. 1. An illustration of graph connectivity.

III. RELATION BETWEEN THE MINIMUM TRANSMISSION RANGE $r_c(n)$ AND n

In this section, we shall derive the key result in this paper, i.e., the relationship between the square of the minimum transmission range $r_c(n)$ and n .

Before we start, we shall introduce a theorem about random geometric graphs by Penrose [10]. Let $\delta_k(n)$ (respectively, $\sigma_k(n)$) denote the minimum transmission range at which a random geometric graph $G(n, r)$ is k -connected (respectively, has minimum node degree k). Penrose has proved the following theorem:

Theorem 1 (Theorem 1.1 in [10]): Consider a random geometric graph $G(n, r(n))$ in \mathbb{R}^d ($d \geq 2$). Given any integer $k > 0$,

$$\lim_{n \rightarrow \infty} Pr\{\delta_k(n) = \sigma_k(n)\} = 1. \quad (1)$$

Remark 1: *Theorem 1* shows that if n is large enough, then with high probability, if one starts with isolated vertices and adds edges connecting the vertices as the transmission range increases, then the resulting graph becomes k -connected at the same time when it achieves $d_{\min}(G) \geq k$.

The connectivity (i.e., $k = 1$) considered in this paper is a special case of *Theorem 1*. Let α be a positive real number close to zero. Let $r_c(n)$ denote the minimum transmission range at which the network becomes connected with a high probability $(1 - \alpha)$; and let $\tau_c(n)$ denote the minimum transmission range at which the network achieves $d_{\min}(G) \geq 1$ with a high probability $(1 - \alpha)$. From *Theorem 1*, we have $r_c(n) \rightarrow \tau_c(n)$ as $n \rightarrow \infty$. For finite but large n , *Theorem 1* and *Remark 1* suggest that $r_c(n) \approx \tau_c(n)$. In fact, for a finite n , $\tau_c(n)$ is a close approximation of $r_c(n)$ when the

probability $Pr\{d_{min}(G) \geq 1\}$ is almost one (see *Theorem 3* of [1]), and this holds even when n is not very large (see Fig. 8-b in [1] where $n = 100$). The above gives us an approach for investigating $r_c(n)$ by approximating it with $\tau_c(n)$ when the probability $Pr\{d_{min}(G) \geq 1\}$ is close to one.

In this paper, we abuse Penrose's theorem a little bit by assuming that the theorem also applies for small values of n and small values of α . As later simulations show that this assumption is a reasonably accurate assumption.

A. Minimum transmission range for connectivity

The probability that the minimum node degree is at least one is given by [1] as

$$Pr\{d_{min}(G) \geq 1\} = (1 - \exp(-n\pi r^2(n)))^n. \quad (2)$$

The derivation of this expression ignores the boundary effect and requires that $n \gg 1$ (e.g., $n = 25, 50$) and $\pi r^2(n) \ll 1$ so that the uniform distribution for the node distribution can be approximated by a homogeneous Poisson point process [1], [10], [11], [13, pp. 39], and the probability that one node having i neighbors can be regarded as almost independent of the probability that any other node having j neighbors.

Because $\tau_c(n)$ is the minimum transmission range at which the network achieves $d_{min}(G) \geq 1$ with a high probability $(1 - \alpha)$, from Eq. 2, we have

$$(1 - \exp(-n\pi\tau_c^2(n)))^n = 1 - \alpha. \quad (3)$$

Now we can seek more insight into the relation between $\tau_c^2(n)$ and n . First, we present the following *Claim 1* we shall use below in our derivation.

Claim 1: For any fixed real number $\alpha \in (0, 1)$, given $\tau_c(n)$ satisfying Eq. 3. Then, the following holds:

$$\lim_{n \rightarrow \infty} \exp(-n\pi\tau_c^2(n)) = 0. \quad (4)$$

Proof: From Eq. 3, we have

$$\exp(-n\pi\tau_c^2(n)) = 1 - (1 - \alpha)^{\frac{1}{n}}. \quad (5)$$

Hence, for any fixed $\alpha \in (0, 1)$, it can be obtained that

$$\begin{aligned} \lim_{n \rightarrow \infty} \exp(-n\pi\tau_c^2(n)) &= \lim_{n \rightarrow \infty} \left(1 - (1 - \alpha)^{\frac{1}{n}}\right) \\ &= 1 - \lim_{n \rightarrow \infty} (1 - \alpha)^{\frac{1}{n}} \\ &= 0. \end{aligned}$$

■

Based on Eq. 3 and *Claim 1*, we have the following *Theorem 2*.

Theorem 2: For any fixed real number $\alpha \in (0, 1)$, given $\tau_c(n)$ satisfying Eq. 3. Then, the following holds:

$$\tau_c^2(n) = \frac{\log n}{n\pi} + \frac{-\log(-\log(1 - \alpha))}{n\pi} + \frac{\log\left(1 + \frac{f(n)}{2(1 - \xi)^2}\right)}{n\pi}, \quad (6)$$

where $f(n) := \exp(-n\pi\tau_c^2(n))$, and ξ is some real number satisfying $0 < \xi < f(n)$.

Proof: Define $f(n) := \exp(-n\pi\tau_c^2(n))$. It is straightforward that $0 < f(n) < 1$, and from *Claim 1*, $f(n) \rightarrow 0$ as $n \rightarrow \infty$. From Eq. 3, we have

$$\log(1 - f(n)) = \frac{1}{n} \log(1 - \alpha). \quad (7)$$

Let us consider the left hand side term of Eq. 7 as a function of $f(n)$. It can be expressed as a Taylor series at zero, i.e.,

$$\log(1 - f(n)) = -f(n) - \frac{f^2(n)}{2(1 - \xi)^2}, \quad (8)$$

where $0 < \xi < f(n)$, and the last term on the right hand side is the *Lagrange remainder term*.

From Eq. 7 and Eq. 8, we have

$$-f(n) - \frac{f^2(n)}{2(1 - \xi)^2} = \frac{1}{n} \log(1 - \alpha). \quad (9)$$

Substituting $f(n) = \exp(-n\pi\tau_c^2(n))$ into Eq. 9, it can be obtained that

$$n \exp(-n\pi\tau_c^2(n)) \left(1 + \frac{f(n)}{2(1 - \xi)^2}\right) = -\log(1 - \alpha). \quad (10)$$

After some manipulations on Eq. 10, we have

$$n\pi\tau_c^2(n) = \log n - \log(-\log(1 - \alpha)) + \log\left(1 + \frac{f(n)}{2(1 - \xi)^2}\right), \quad (11)$$

which immediately yields Eq. 6. ■

Remark 2: Because $0 < \xi < f(n)$, $\xi \rightarrow 0$ as $f(n) \rightarrow 0$. Hence, we have

$$\lim_{n \rightarrow \infty} \log\left(1 + \frac{f(n)}{2(1 - \xi)^2}\right) = 0.$$

When $n \gg 1$ (e.g., $n = 25, 50$) and α is close to zero (e.g., $\alpha = 0.05, 0.01$), the third term can be ignored compared with the first term and the second term on the right hand side of Eq. 11.

Because when $n \gg 1$ and α is close to zero, $f(n) \approx \frac{\alpha}{n}$ from Eq. 5 and the definition of $f(n)$. Then,

$$\begin{aligned} \log\left(1 + \frac{f(n)}{2(1 - \xi)^2}\right) &< \frac{f(n)}{2(1 - \xi)^2} \\ &< \frac{f(n)}{2(1 - f(n))^2} \approx \frac{\alpha/n}{2(1 - 2\alpha/n)} \\ &= \frac{\alpha}{2(n - 2\alpha)} \approx \frac{\alpha}{2n}. \end{aligned} \quad (12)$$

The first term and the second term on the right hand side of Eq. 11 are $\log n$ and $-\log(-\log(1 - \alpha))$ respectively. When $\alpha \in (0, 1 - \frac{1}{e})$, the second term is positive, strictly monotonically decreasing with α and $(-\log(-\log(1 - \alpha))) \rightarrow \infty$ as $\alpha \rightarrow 0$. When α is close to zero, we have $-\log(-\log(1 - \alpha)) \approx -\log \alpha$. Comparing $\log n$ and $(-\log \alpha)$ with Eq. 12, one can readily obtain that the third term can be ignored.

Based on *Theorem 2* and *Remark 2*, we can obtain that $\tau_c^2(n)$ is approximately determined by the first term and the second term on the right hand side of Eq. 6, i.e.,

$$\tau_c^2(n) \approx \frac{\log n}{n\pi} + \frac{-\log(-\log(1 - \alpha))}{n\pi}.$$

As indicated in the previous analysis based on the results in [1] and [10], $\tau_c(n)$ is a close approximation of $r_c(n)$ when the probability $Pr\{d_{min}(G) \geq 1\}$ is close to one. From Eq. 2 and Eq. 3, it is straightforward that when α is close to zero, the probability $Pr\{d_{min}(G) \geq 1\}$ is close to one. Therefore, we have

$$r_c^2(n) \approx \tau_c^2(n) \approx \frac{\log n}{n\pi} + \frac{-\log(-\log(1-\alpha))}{n\pi}. \quad (13)$$

From Eq. 13, we calculate the analytical values of the average node degree at which the network is connected with probability $(1-\alpha)$, which are shown in Table. I. One can find that for $n = 25 \sim 300$, if the average node degree is set to be some constant between 6 and 10, the network becomes connected with a high probability. The result is consistent with those in [3].

TABLE I
ANALYTICAL VALUES OF THE MINIMUM AVERAGE NODE DEGREE.

α	n					
	25	50	75	100	200	300
0.005	8.51	9.21	9.61	9.90	10.59	11.00
0.01	7.82	8.51	8.92	9.21	9.90	10.30
0.05	6.19	6.88	7.29	7.58	8.27	8.67

Based on Eq. 13, we are able to state the result as follows.

Theorem 3: Given $0 < \alpha \ll 1$, define $r_c(n)$ as

$$r_c(n) := \inf\{r > 0 : Pr\{G(n, r) \text{ is connected}\} = 1 - \alpha\}.$$

Then for small n (e.g., $n = 25 \sim 300$), $r_c^2(n)$ grows approximately linearly with $\frac{1}{n}$; as n goes to infinity, $r_c^2(n)$ grows like $\frac{\log n}{n}$, in other words, the average node degree must grow with $\log n$.

Proof: For any fixed small n , because when $\alpha \in (0, \frac{1}{e})$, $-\log(-\log(1-\alpha))$ is positive, strictly monotonically decreasing with α and $(-\log(-\log(1-\alpha))) \rightarrow \infty$ as $\alpha \rightarrow 0$, there exists a constant α_n such that $-\log(-\log(1-\alpha)) \gg \log n$ for all $\alpha < \alpha_n$. Hence, from Eq. 13, $r_c^2(n)$ grows approximately linearly with $\frac{1}{n}$.

Similarly, for any fixed α ($0 < \alpha \ll 1$), there exists a constant $N(\alpha)$ such that $\log n \gg -\log(-\log(1-\alpha))$ for all $n > N(\alpha)$. Hence, from Eq. 13, $r_c^2(n)$ grows like $\frac{\log n}{n}$, namely, $n\pi r_c^2(n)$ grows like $\log n$. Therefore, for sufficiently large n , if the average node degree is fixed to be a constant, then the network will almost surely be disconnected. ■

Remark 3: *Theorem 3* is consistent with the conclusion stated in [2], [3] for small n and in [4], [5], [6] for sufficiently large n .

IV. SIMULATION

In this section, we perform simulations to verify our theoretical analysis. We programmed a tool in C++ for the simulations. In the simulations, a total of n nodes are randomly and independently distributed in a unit square according to a uniform distribution. All nodes have the same transmission range. For each given n , we calculate $\tilde{r}_c(n)$ and $\tilde{d}_{avg}(G)$ in simulations. Here $\tilde{r}_c(n)$ represents the simulated minimum

transmission range at which the network with n nodes is connected with a high probability $(1-\alpha)$; and $\tilde{d}_{avg}(G) = n\pi\tilde{r}_c^2(n)$ is the simulated average node degree at which the network with n nodes is connected with a high probability $(1-\alpha)$.

We have used the *toroidal distance metric* [1] to remove the impact of the boundary effect on the simulation results.

Firstly, we verify the result given by Eq. 13. Fig. 2 shows the square of $r_c(n)$ versus the total number of nodes n for different values of α . The analytical results are calculated from Eq. 13. We can see that the analytical results and the simulation results match very well, which means that $\tau_c(n)$ is a close approximation of $r_c(n)$ when $Pr\{d_{min}(G) \geq 1\}$ is close to one. Hence, the previous approximation (i.e., $r_c(n) \approx \tau_c(n)$) is a reasonable assumption. We can also see that the smaller α is, the better the simulation results agree with the analytical results, since the approximation of $r_c(n)$ by $\tau_c(n)$ becomes more accurate if the probability $Pr\{d_{min}(G) \geq 1\}$ is much closer to one, i.e., α is much closer to zero.

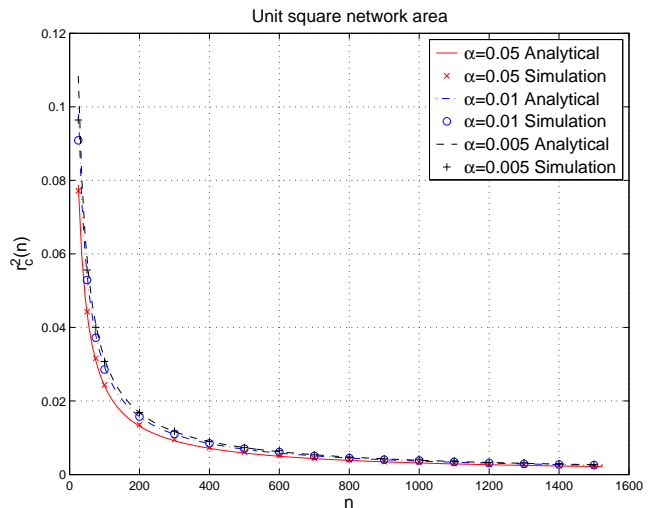


Fig. 2. $r_c^2(n)$ versus n . $r_c(n)$ is the minimum transmission range at which the network is connected with probability $(1-\alpha)$.

Fig. 3 shows the simulation results of the connectivity probability versus the average node degree for different values of n . In the simulations, the average node degree is calculated as $n\pi\tilde{r}_c(n)$. From Fig. 3, one can find that for $n = 25 \sim 200$, the average node degree, at which the network is connected with a high probability, is a constant in the range between 6 to 10. This is consistent with the analytical values shown in Table. I and the conclusion presented in [3].

Fig. 4 shows the relationship between the squared inverse of $r_c(n)$ and n for different values of α and for small values of n . n varies from 25 to 300. The analytical results are also calculated using Eq. 13. We can see that $\frac{1}{r_c^2(n)}$ grows approximately linearly with n , namely, $r_c^2(n)$ grows approximately linearly with $\frac{1}{n}$. In addition, the smaller α is, the better the relationship is. Fig. 5 shows the square of $r_c(n)$ versus $\frac{\log n}{n}$ for different values of α and for large values of n .

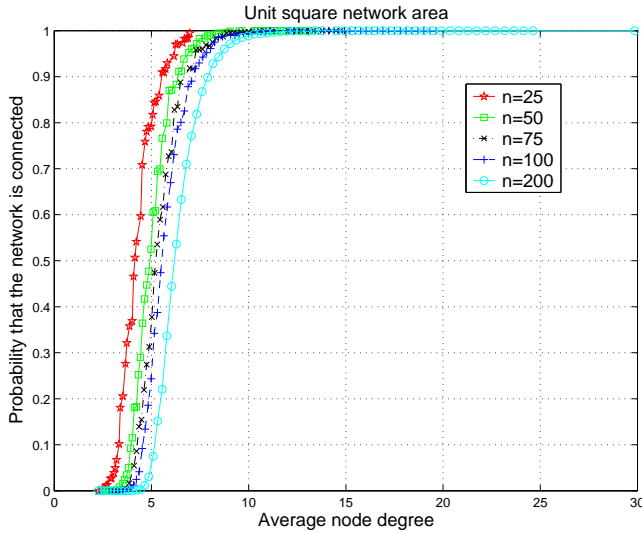


Fig. 3. Simulation results: connectivity probability versus average node degree. The average node degree is calculated as $n\pi\tilde{r}_c^2(n)$.

n varies from 900 to 1500. The analytical results are calculated using Eq. 13. One can find that the square of $r_c(n)$ grows approximately linearly with $\frac{\log n}{n}$.

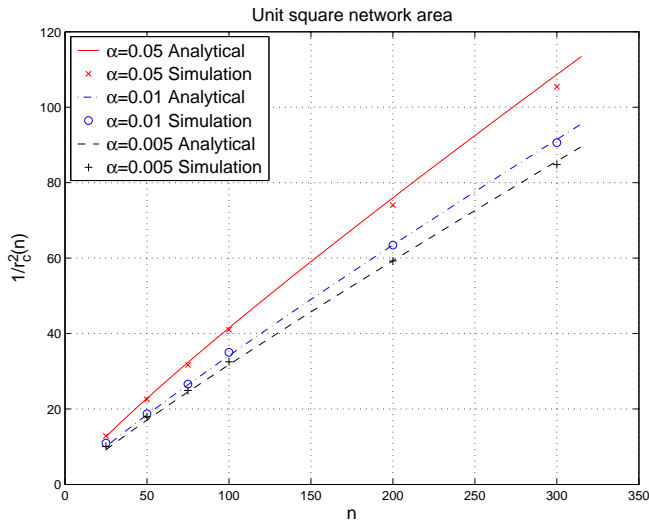


Fig. 4. $\frac{1}{r_c^2(n)}$ versus n for $n = 25 \sim 300$. $r_c(n)$ is the minimum transmission range at which the network is connected with probability $(1-\alpha)$.

V. CONCLUSION

In this paper, we investigated the characteristics of the minimum transmission range $r_c(n)$ at which the network is connected with a high probability. Both theoretical analysis and simulations were presented. It was shown that for small values of n , e.g., $n = 25 \sim 300$, $r_c^2(n)$ grows approximately linearly with $\frac{1}{n}$, but as n goes to infinity, $r_c^2(n)$ must grow like $\frac{\log n}{n}$. In other words, for small values of n , there exists a constant, called “magic number” in some papers; when the average node degree is equal to that constant, the network

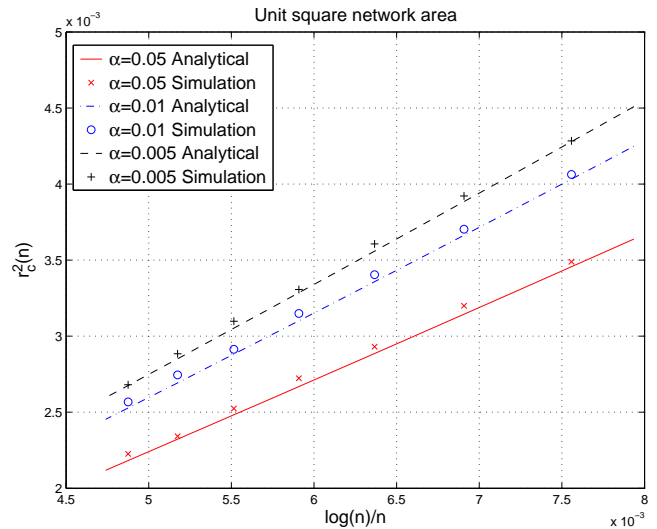


Fig. 5. $r_c^2(n)$ versus $\frac{\log n}{n}$ for $n = 900 \sim 1500$. $r_c(n)$ is the minimum transmission range at which the network is connected with probability $(1-\alpha)$.

becomes connected with a high probability. However, when n is sufficiently large, the average node degree should not remain constant but grow like $\log n$, and there is no such thing as a “magic number”. The results of this paper are very useful for network design and may guide strategies for power control and routing in wireless sensor/ad hoc networks.

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