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On Cooperative Communication in Ad-Hoc Networks: The Case for Uncoordinated Location-Aware Retransmission Strategies

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***Abstract*—*Cooperative communication* methods in wireless networks, ranging from relaying by a common neighbor over a single wireless hop to opportunistic routing at the network layer, have been shown in recent years to offer significant performance gains over traditional approaches that ignore the broadcast nature of the wireless medium, and are particularly valuable in environments prone to channel shadowing and fading, such as mobile ad-hoc networks. A common feature of various cooperative methods proposed in the literature is the *coordination* required to discover the available neighbors and determine the optimal one(s) to be involved in the cooperation. However, the overhead cost of such coordination may be prohibitive in networks with highly dynamic topology (e.g. due to high mobility), as the discovery and negotiation overheads may negate much of the cooperation gains. Accordingly, we present the case for *uncoordinated* cooperative retransmission, where a node overhearing a frame may retransmit it "blindly" without any prior coordination with the transmitter, intended receiver, or any other neighbors in the vicinity. We pose and solve the problem of finding the optimal retransmission probability as a function of location, and characterize the optimal uncoordinated cooperation region through the solution of an integral equation that depends only on the *a priori* node density and wireless propagation model. Through numerical evaluation, we demonstrate that uncoordinated cooperation provides a low-overhead viable alternative, with a frame delivery probability that can even exceed that of coordinated cooperation methods in situations with high noise (or low transmission power) and high node density.**

I. Introduction

The traditional layered design of wireless networks has followed a similar approach to wired networks, where the route between a source and a destination is determined in advance and each node along the route is solely responsible for delivering the respective data frames to its next hop. Hence, the only remedy available against a failed reception by a node's next hop is retransmission by the same node, either of the frame itself (classic ARQ) or of error-correcting codes providing additional redundancy (hybrid ARQ). Clearly, this approach is inflexible in responding to short-term variations of wireless link quality. Recognizing this limitation, the concept of *cooperative communications* has attracted considerable research attention in recent years. Cooperative communication methods take advantage of the broadcast nature of the wireless medium and provide diversity against link fades and outages on the main path, by allowing additional nodes in the vicinity of a route that overhear a transmitted signal to assist in delivering the data to its destination.

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The majority of cooperative methods considered in the literature fall under the category of physical-layer cooperation, or *cooperative relaying*. For a single relay, these range from amplification of the analog source signal directly (*amplify-and-forward*) to retransmission of the frame after its decoding (*decode-and-forward*) [1], [2], or transmission of error-correcting code bits allowing the receiver to decode the frame by combining the signals from both the original source and the relay (*coded cooperation*) [3]. Similar methods have been proposed for multiple-relay cooperation, where the relay signals are multiplexed on orthogonal CDMA [4] or TDMA [5] subchannels. More recently, there has been increasing interest in multiple-relay cooperation methods over a single subchannel, based on *space-time codes* (cf. [6]–[8] and references therein).

Cooperative methods have been considered in a higher-layer (i.e. multi-hop) context as well. In network-layer cooperation, commonly known as *opportunistic routing*, a frame is broadcast at every hop and the next-hop node to take responsibility of forwarding it further is chosen opportunistically among the nodes where the broadcast has actually been received; this provides *path diversity* to the route of each data frame (rather than diversity for a single link). Several alternatives have been proposed for the coordination process among the receiving nodes to choose the next hop. Selection Diversity Forwarding (SDF) [9] uses explicit acknowledgments to the broadcasting node who then selects and notifies the best next hop, and Geographic Random Forwarding (GeRaF) [10] follows a similar procedure using RTS/CTS control frames rather than ACKs of the actual data frame. In Extremely Opportunistic Routing (ExOR) [11], instead of explicit ACK or CTS exchanges, nodes simply wait for a random period before forwarding the frame, or discard the frame if they overhear it being forwarded by another node in the meantime.

A common feature of these and similar cooperative tech-

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niques in existing literature is the *coordination* required among the participating neighbors. In cooperative relaying, the initial set-up generally involves the discovery of neighbors in the vicinity and the selection of the best one(s) whose cooperation will maximize the performance improvement. For multiple-relay methods, further coordination is required to allocate the separate subchannels or the space-time codes for the relayed signals. The overheads associated with such coordination mean that existing cooperative methods are suitable mostly for networks with static or relatively stable topologies, such as wireless mesh networks, so that the respective overheads can be amortized over a large number of frames. When the topology is highly dynamic — such as in ad hoc networks with highly mobile nodes, or sensor networks with unpredictable sleep patterns — these overheads are incurred frequently and can negate much of the potential performance gains. Furthermore, when opportunistic routing is employed, the coordination process of the next-hop decision imposes an additional per-frame overhead, either in terms of the explicit control messages exchange or, as in ExOR [11], traded for a higher expected latency of each frame, which makes it impractical for highly dynamic networks.

Motivated by the above observation, we set out to explore the option of *uncoordinated* cooperation, where a node overhearing a frame may take a cooperative action — namely, immediately retransmit the frame — without any prior coordination with other nearby nodes, and even without being aware of their existence (apart from the transmitter and receiver of the frame). Due to the lack of coordination, such cooperative retransmissions are prone to *collision* at the receiver if made by several nodes simultaneously. Accordingly, we define the *cooperation strategy* of a node as a probability value to make the cooperative retransmission. Our model assumes that every network node knows its own location at all times, and the cooperation strategy may therefore depend on the location of the node itself, the locations of the transmitter and receiver (which are assumed to be included in the header of each frame), and general *a priori* assumptions about the spatial distribution of other nodes (e.g. the density of a point Poisson process) and the wireless propagation characteristics of the environment (e.g. the transmitted power attenuation as a function of distance). However, the strategy may not depend on the actual locations of any other nodes, as we explicitly assume an uncoordinated cooperation that does not rely on a prior neighbor discovery phase.

Our contribution in this paper is twofold. First, we present a generic analysis of finding the optimal cooperation strategy, i.e. one that maximizes the probability of successful decoding by the receiver. Unlike most optimizations encountered in related studies, which involve a finite or at most a countable set of variables, the optimization in our case is over a function of a continuous location, and is therefore tackled using functional optimization methods. It turns out that a deterministic optimal strategy always exists, defined by an optimal *cooperation region* such that neighbors within that region should always retransmit an overheard frame with probability 1, while neighbors outside that region should not retransmit. This optimal region is characterized by a solution of an integral equation that depends only on the *a priori* spatial distribution of the nodes and the signal propagation model. Second, we conduct a numerical evaluation for a variety of scenarios based on a realistic propagation model, and demonstrate that the frame delivery probability of our proposed method is competitive with other methods, especially in high-noise (or low transmission power) environments with high node density, while not incurring any coordination overheads.

The rest of this paper is structured as follows. Section II describes our network model and formulates the functional optimization problem, and its analytical solution is presented in Section III. The numerical study that demonstrates the superior performance of the proposed method is presented in Section IV. Finally, Section V concludes the paper.

II. Model and Problem Formulation

A. Model description

Consider an ad hoc network in a region V in which nodes are distributed according to a generalized point Poisson process with bounded density $\rho(v), v \in V$. The density function can be either two- or three-dimensional, and may or may not be constant throughout the region. We assume that this function is time-stationary, which, for example, is generally the case with random direction mobility models [12]. We assume a localization mechanism such that each node is aware of its own location. Furthermore, when a connection is established between two nodes, the locations of both endpoints are included in all subsequently exchanged frames. Therefore, when a node overhears a frame transmitted in its vicinity, it becomes aware of the locations of that frame's sender and receiver. Other than that, we assume no knowledge of the network topology; thus, there is no neighbor discovery protocol in the background, and moveover, if any location information was available in other frames overheard in the past, we assume that nodes either do not keep track of it, or that any such information becomes stale very quickly. Thus, no node (including the sender) knows the number of any other nodes that may potentially overhear the frame, their locations, or their channel conditions.

The subsequent discussion considers cooperation strategies in the context of a generic single frame between a given sender and receiver. If the frame is not received successfully (as is detected by a lack of acknowledgment), a node that had overheard it will make a single immediate retransmission with probability $\tau(v)$, where v is the location of the node. We refer to $\tau(v)$ as the *strategy* function, and the goal of our analysis will be to find the function that maximizes the probability of successful delivery to the receiver. Thus, we deliberately focus our attention on the simplest possible uncoordinated cooperation strategy, with just a single cooperative retransmission attempt that immediately follows the original frame. Our analysis of this strategy and its performance will present the case for uncoordinated cooperation and its potential benefits, and establish the required fundamental framework for investigating further possible enhancements, such as carrier sensing and/or random waiting before the retransmission, or coded cooperative transmission of error-correcting bits rather than the frame itself. The further study of such enhancements and their potential benefits is left for future work.

We proceed to introduce the notation to describe the wireless environment for a given sender and receiver. We assume that the radio channel between any two nodes is either "good" (i.e. a transmission over that channel will be decoded successfully, barring interference from other simultaneous transmissions) or "bad" (a frame transmitted over the channel will not be decoded) for the duration of a given frame, and that the channel conditions in different locations are independent of each other. Accordingly, we define the following location-dependent probabilities for a generic location $v \in V$:

- $P_{sn}(v)$ is the *a priori* probability that the *interim channel* (i.e. the channel between the *s*ource and a *n*ode located at v) is good, i.e. the node overhears a transmitted frame;
- $P_{nd}(v)$ is the *a priori* probability that the *relay channel* (i.e. the channel between a *n*ode at location v and the *d*estination) is good.

We emphasize that the functions $P_{sn}(v)$ and $P_{nd}(v)$ are used to model the wireless environment; in other words, they are conditional probabilities, describing the channel quality in a location conditioned on a node indeed existing in that location. We point out that this notation is extremely generic, as $P_{sn}(v)$ and $P_{nd}(v)$ can be arbitrary. For example, they may be based on any popular distance-based propagation model (e.g. a unit disk model, log-normal shadowing model, etc); however, our analysis does not require any particular structure of these functions, or even that they are monotonic in the channel distance. The only assumption we make is that these *a priori* functions are known to the nodes in advance, and any node can thus evaluate them for its own location.

We now describe our assumptions about interference among multiple simultaneous retransmissions. We assume that, if two or more retransmissions are made over respectively good relay channels, they will always collide and result in mutual destruction. If there is one transmission over a good relay channel with other transmissions taking place simultaneously over bad ones, we distinguish between the following two cases.

- *Case I*: the transmission is successful (i.e. transmissions over bad channels does not cause interference). Broadly, this corresponds to the case where the underlying cause for bad channels is signal blockage (e.g. by a physical obstacle), leading to low received signal power throughout the duration of the transmission. We henceforth refer to this case as "non-interfering bad channels".
- *Case II*: the transmission is unsuccessful (i.e. a transmission over a bad channel destroys another good one at the receiver). This happens when the signal(s) over the bad channel(s) are received with sufficiently high power to cause interference, despite not being able to be decoded, and can be caused by a variety of wireless propagation phenomena (e.g. multipath fading, shadowing, Doppler shifts, or a combination of the above). To keep the discussion generic, we simply refer to this case as "interfering bad channels".

For the sake of clarity of presentation, we conduct the analysis in section III for each of these "pure" cases first, i.e. assuming that all bad relay channels in the network are of the same kind (either interfering or non-interfering). Subsequently, we extend the analysis to a mixed scenario where both kinds of bad channels can coexist in the network, governed by a location-dependent interference probability of its own.

B. Problem formulation

Consider the success probability of a cooperative retransmission attempt in the above model. Since all retransmissions immediately follow an unsuccessful one by the source and are uncoordinated, collisions are possible, resulting in an ultimate failure even though some of the nodes had good interim and relay channels. Therefore, we define the optimization problem of finding the retransmission strategy $\tau(v)$ to maximize the overall success probability of the retransmission. Clearly, since $\tau(v)$ is a function of a (continuous) location v, the corresponding optimization is defined and tackled using functional analysis methods.

We now derive the target functionals (namely, the probabilities of successful retransmission) for each of the two cases of bad channels, as defined in the previous subsection. To that end, we divide the space V into a lattice of small cubes of size Δv and assume that nodes can only be placed in the centres of the cubes (i.e. in discrete locations), such that the probability of a node existing in the cube containing location v is $\rho(v)\Delta v + o(\Delta v)$.

For case I, since retransmissions over bad channels do not cause interference, the only requirement for a retransmission to be successful is that there is precisely one node which (1) overhears the frame (i.e. has a good interim channel), (2) makes a retransmission, *and* (3) has a good relay channel, while all other nodes fail at least one of these three conditions:

$$P^I_{suc} = \sum_v P_{sn}(v)\tau(v)P_{nd}(v)\rho(v)\Delta v \cdot \prod_{v' \neq v} [1 - P_{sn}(v')\tau(v')P_{nd}(v')\rho(v')\Delta v] \quad (1)$$

As $\Delta v \to 0$, we note that

$$\lim_{\Delta v \to 0} \prod_{v' \neq v} [1 - P_{sn}(v')\tau(v')P_{nd}(v')\rho(v')\Delta v] =$$
$$\lim_{\Delta v \to 0} \exp\left\{ \sum_{v' \neq v} \log [1 - P_{sn}(v')\tau(v')P_{nd}(v')\rho(v')\Delta v] \right\} =$$
$$\lim_{\Delta v \to 0} \exp\left\{ - \sum_{v' \neq v} P_{sn}(v')\tau(v')P_{nd}(v')\rho(v')\Delta v \right\} =$$
$$\exp\left\{ - \int_{v' \in V} P_{sn}(v')\tau(v')P_{nd}(v')\rho(v')dv' \right\} \quad (2)$$

becomes independent of v (reflecting the infinitesimal impact of a single excluded point in a continuous space). Therefore, we can drop the distinction between v and v', and the expression for the success probability (1) becomes

$$P_{suc}^{I} = \int_{v \in V} P_{sn}(v)\tau(v)P_{nd}(v)\rho(v)dv \cdot \exp\left\{-\int_{v \in V} P_{sn}(v)\tau(v)P_{nd}(v)\rho(v)dv\right\}. \quad (3)$$

Remark. Expression (3) can also be derived using the theory of *marked Poisson processes* [13], by defining a 'marked' node as one that has good interim and relay channels and transmits (i.e. a node is marked with probability $P_{sn}(v)\tau(v)P_{nd}(v)$), and concluding that the number of such nodes is Poisson-distributed with a parameter of $\int_{v \in V} P_{sn}(v)\tau(v)P_{nd}(v)\rho(v)dv$. Then, expression (3) corresponds simply to the probability of having exactly one marked node in V. Nevertheless, we follow the above direct derivation as it can be readily extended to the other case below.

For the case of interfering bad channels (case II), a successful cooperative retransmission requires one node that has a good interim channel, chooses to retransmit, and has a good relay channel, while all other nodes must either not overhear the frame or not retransmit it; unlike case I, the state of the relay channels of the other nodes is immaterial. Using a similar division of space as before, we arrive at the expression

$$P_{suc}^{II} = \sum_{v} P_{sn}(v)\tau(v)P_{nd}(v)\rho(v)\Delta v \cdot \prod_{v' \neq v} [1 - P_{sn}(v')\tau(v')\rho(v')\Delta v] \quad (4)$$

which, in the limit of $\Delta v \to 0$, becomes

$$P_{suc}^{II} = \int_{v \in V} P_{sn}(v)\tau(v)P_{nd}(v)\rho(v)dv \cdot \exp\left\{-\int_{v \in V} P_{sn}(v)\tau(v)\rho(v)dv\right\}. \quad (5)$$

Thus, the difference between the two cases is that $P_{nd}(v)$ is omitted from the integral inside the exponent in the latter.

We now consider the possibility that it is the source itself, rather than one of the cooperative neighbor nodes, that makes an immediate retransmission. To that end, we denote by P_{sd} the probability of the direct channel between the source and destination to become good again, immediately after a failed original transmission. Note that, due to the possible temporal correlation of channel quality, P_{sd} can (and typically will) be lower than the normal *a priori* probability of a good channel between the source and destination locations. We assume P_{sd} is independent of the state of all other channels.

For a retransmission by the source to be collision-free, the same requirement holds as before for retransmissions of other nodes (namely, no simultaneous retransmissions over good relay channels in case I, and no simultaneous retransmissions at all in case II). If the retransmission probability (i.e. strategy) of the source node is τ_s, the respective success probability expressions become

$$P_{suc}^{I} = \left[(1-\tau_s P_{sd})\int_{v \in V} P_{sn}(v)\tau(v)P_{nd}(v)\rho(v)dv + \tau_s P_{sd}\right] \cdot \exp\left\{-\int_{v \in V} P_{sn}(v)\tau(v)P_{nd}(v)\rho(v)dv\right\} \quad (6)$$

and

$$P_{suc}^{II} = \left[(1-\tau_s)\int_{v \in V} P_{sn}(v)\tau(v)P_{nd}(v)\rho(v)dv + \tau_s P_{sd}\right] \cdot \exp\left\{-\int_{v \in V} P_{sn}(v)\tau(v)\rho(v)dv\right\}. \quad (7)$$

To summarize, we state the functional optimization problem that is the subject of the analysis in the next section:

Maximize P_{suc} (given by (6) or (7), respectively)
subject to $0 \le \tau_s \le 1$ and $0 \le \tau(v) \le 1$, $v \in V$.

III. Optimal Retransmission Strategy Analysis

In this section, we first solve the functional optimization problem defined above for each of the 'pure' cases separately. We then consider the mixed case where interfering and non-interfering bad channels can coexist in the same network.

A. Case I: non-interfering bad channels (signal blockage)

We define $\lambda \triangleq \int_{v \in V} P_{sn}(v)\tau(v)P_{nd}(v)\rho(v)dv$ for convenience, and rewrite expression (6) as

$$P_{suc}^{I} = \tau_s P_{sd}(1-\lambda)\exp(-\lambda) + \lambda\exp(-\lambda). \quad (8)$$

Clearly, the entire dependence of the success probability on $\tau(v)$ is captured through λ. Thus, the functional optimization reduces to a simple optimization problem of two variables, with corresponding constraints of

$$0 \le \tau_s \le 1 \quad (9)$$

$$0 \le \lambda \le \lambda^{\max} \triangleq \int_{v \in V} P_{sn}(v)P_{nd}(v)\rho(v)dv, \quad (10)$$

where $\lambda^{\max}$ is the largest possible value of λ that can be obtained by setting $\tau(v) = 1$ everywhere. Once the optimal λ is found, a degree of freedom remains for the choice of any particular function $\tau(v)$ that corresponds to that value of λ.

We observe that (8) is linear in τ_s; hence, its maximum will always be attained at either $\tau_s = 0$ or $\tau_s = 1$. Furthermore, if the maximum is attained with $\tau_s = 0$ and not $\tau_s = 1$, then the coefficient of τ_s must be negative, implying $\lambda > 1$. However, expression (8) in that case becomes simply $P_{suc}^{I} = \lambda\exp(-\lambda)$, which, of course, is maximized at $\lambda = 1$ — a contradiction. Therefore, we conclude that the maximum P_{suc}^{I} is always attained with $\tau_s = 1$.

It remains to find the optimal value of λ. Since $\tau_s = 1$ in the optimum, expression (8) becomes

$$P_{suc}^{I} = [P_{sd} + (1-P_{sd})\lambda]\exp(-\lambda). \quad (11)$$

A straightforward analysis of this expression easily leads to the following conclusions:

- If $P_{sd} \ge 0.5$, then (11) is monotonically decreasing in λ; hence, the optimal value is $\lambda = 0$ (no retransmissions by any nodes but the source), resulting in $P_{suc} = P_{sd}$;
- If $P_{sd} < 0.5$, expression (11) increases with λ up to a maximum at $\lambda = \frac{1-2P_{sd}}{1-P_{sd}}$. Therefore, the optimum is attained at $\lambda = \min\left(\frac{1-2P_{sd}}{1-P_{sd}}, \lambda^{\max}\right)$.

B. Case II: interfering bad channels

Once again, due to the linearity of expression (7) in τ_s, the maximum value of P_{suc} must be attained at either $\tau_s = 0$ or $\tau_s = 1$. We begin by considering the case of $\tau_s = 0$ first.

If $\tau_s = 0$, then the probability of successful retransmission is given by (5). We tackle the maximization of (5) by considering it as the limit of a discrete problem. Accordingly, we again divide the space V into a lattice of small cubes of size Δv as above; thus, the probability of successful retransmission is given by (4). We now calculate the partial derivative of (4) with respect to a single variable $\tau(v_0)$, corresponding to a particular location v_0:

$$\frac{\partial P_{suc}^{II}}{\partial \tau(v_0)} = P_{sn}(v_0) P_{nd}(v_0) \rho(v_0) \Delta v \cdot \exp\left\{ -\sum_{v' \neq v_0} P_{sn}(v')\tau(v')\rho(v')\Delta v \right\} + \sum_{v'' \neq v_0} P_{sn}(v'')\tau(v'')P_{nd}(v'')\rho(v'')\Delta v \cdot [-P_{sn}(v_0)\rho(v_0)\Delta v] \cdot \exp\left\{ -\sum_{v' \neq v''} P_{sn}(v')\tau(v')\rho(v')\Delta v \right\} \quad (12)$$

Therefore, as $\Delta v \to 0$,

$$\lim_{\Delta v \to 0} \frac{1}{\Delta v} \frac{\partial P_{suc}^{II}}{\partial \tau(v_0)} = P_{sn}(v_0)\rho(v_0) \left[P_{nd}(v_0) - \int_{v \in V} P_{sn}(v)\tau(v)P_{nd}(v)\rho(v)dv \right] \cdot \exp\left\{ -\int_{v \in V} P_{sn}(v)\tau(v)\rho(v)dv \right\}. \quad (13)$$

We observe that the sign of the derivative is determined by the sign of the part in brackets on the right-hand side of (13),

$$\left[P_{nd}(v) - \int_{v \in V} P_{sn}(v)\tau(v)P_{nd}(v)\rho(v)dv \right]. \quad (14)$$

Furthermore, we note that (14) is independent of the choice of $\tau(v_0)$ itself, i.e. varying $\tau(v)$ in a single point $v = v_0$ has infinitesimal impact on the derivative (and its sign). We conclude that P_{suc}^{II} is maximized if $\tau(v)$ is set to 1 wherever the derivative is positive, and to 0 wherever it is negative. Thus, we have established the following lemma.

Lemma 1. *If the function $\tau(v)$ satisfies*

$$\begin{cases} \tau(v) = 1 & P_{nd}(v) > \int_{v \in V} P_{sn}(v)\tau(v)P_{nd}(v)\rho(v)dv \\ \tau(v) = 0 & P_{nd}(v) < \int_{v \in V} P_{sn}(v)\tau(v)P_{nd}(v)\rho(v)dv \end{cases} \quad (15)$$

then it maximizes P_{suc}^{II} for $\tau_s = 0$ (as given by (5)*).*

Unfortunately, Lemma 1 does not lead to an explicit solution of $\tau(v)$, since the condition is stated in a recursive fashion. The following theorem provides the final link to that end.

Theorem 1. *Expression* (5) *is maximized by a function $\tau(v)$ that satisfies*

$$\begin{cases} \tau(v) = 1 & P_{nd}(v) > T \text{ or } P_{nd}(v) = T \text{ and } v \in R_T \\ \tau(v) = 0 & P_{nd}(v) < T \text{ or } P_{nd}(v) = T \text{ and } v \notin R_T \end{cases} \quad (16)$$

where the threshold T and region $R_T \subseteq \{v | P_{nd}(v) = T\}$ solve the integral equation

$$T = \int_{\{v|P_{nd}(v) > T\} \bigcup R_T} P_{sn}(v)P_{nd}(v)\rho(v)dv. \quad (17)$$

Proof: If $\tau(v)$ satisfies the conditions in (16), then, by the definition of T and R_T, it is easily confirmed that it satisfies the conditions of Lemma 1.

It remains to show that the integral equation (17) has a solution. Indeed, we observe that the integral $I(t) \triangleq \int_{\{v|P_{nd}(v)>t\}} P_{sn}(v)P_{nd}(v)\rho(v)dv$ is positive and monotonously decreasing in t (since a larger t implies a smaller integration domain). Consequently, there exists a unique finite T that is the supremum of $\{t | t \leq I(t)\}$ (or, equivalently, the infimum of $\{t | t \geq I(t)\}$). It is possible that $I(t)$ is discontinuous at $t = T$, such that $I(T^+) = I(T) < T$ and $I(T^-) > T$. Accordingly, we define $\Delta I \triangleq I(T^-) - I(T) = T \cdot \int_{\{v|P_{nd}(v)=T\}} P_{sn}(v)\rho(v)dv$. If $\Delta I = 0$ (no discontinuity), then, obviously, equation (17) is satisfied for any R_T. Otherwise, since $P_{sn}(v)\rho(v)$ is bounded, there must exist a region $R_T \subseteq \{v | P_{nd}(v) = T\}$ such that $T \cdot \int_{R_T} P_{sn}(v)\rho(v)dv = T - I(T)$, and that will therefore be a solution of (17). ■

Remark. Theorem 1 does not claim that the maximizing solution as given by (16) is *unique*. In fact, as is always the case when the optimization target is defined by an integral, there cannot be a unique maximizing strategy, since the integral is not impacted by any variation of the solution over any set of measure 0.

We make several comments about the practical implications of Theorem 1. First, equation (17) can be solved numerically by a search for the solution of the equation $I(t) - t = 0$ (e.g. using Newton's method), followed by a separate search for the region R_T if the first search fails due to a discontinuity of $I(t)$. Moreover, we underscore that such a discontinuity is not possible under many common propagation models, where the set $\{v | P_{nd}(v) = T\}$ has measure zero for any T (e.g., in a model with distance-based path loss and log-normal shadowing, any given probability value of the received power to be above the threshold necessary for decoding corresponds to a sphere (in 3-D) or a circle (in 2-D), which has measure zero); consequently, the extra complication of finding the region R_T does not arise. Finally, we emphasize that finding the solution of equation (17) is not a process that needs to be undertaken by every node after every overheard frame. Rather, it only needs to be solved once in advance based on the *a priori* density and propagation model. Moreover, for most common propagation models where the function $P_{nd}(v)$ depends only on the distance to the destination, the optimal threshold of $P_{nd}(v)$ can be translated to a corresponding threshold in terms of distance, leaving every node only to check for every overheard frame whether its distance to that frame's destination is below the threshold.

Finally, we recall that the above analysis has considered the maximization of P_{suc}^{II} for $\tau_s = 0$. We now consider the option of $\tau_s = 1$. Obviously, if $\tau_s = 1$, then P_{suc}^{II} (expression (7)) is maximized with $\tau(v) = 0$ everywhere (indeed, a transmission by any other node will cause a certain collision); hence, in this case, we have $P_{suc}^{II} = P_{sd}$. Accordingly, the optimal cooperative retransmission strategy is defined by Theorem 1 if it results in $P_{suc}^{II} > P_{sd}$; otherwise, the optimal strategy is to have a retransmission by the original source only.

C. Mixed case: non-interfering and interfering bad channels

We conclude this section by considering a mixed scenario, where different relay channels in the same network may fail either due to signal blockage by physical obstacles or due to other wireless propagation impairments; thus, the network may contain a mix of non-interfering and interfering bad channels. To that end, we define the function $P_{sbnd}(v)$ as the probability of the channel from v to the destination to be blocked by a physical obstacle (i.e. be of the non-interfering kind), conditioned on the relay channel at location v indeed being bad. In other words, $P_{sbnd}(v)$ reflects the relative likelihood of the relay channel in a particular location to be affected by signal blockage rather than wireless impairments such as fading, shadowing, Doppler shifts, etc, that do not block the signal power at the receiver. For the direct channel between the source and destination, we denote this probability as P_{sbsd}. As with all other location-dependent probabilities defined in the paper, we assume that the probabilities $P_{sbnd}(v)$ and P_{sbsd} are independent among different locations.

With this definition, the total probability for a transmission from location v to cause interference (assuming another transmission taking place simultaneous over a good relay channel from a node elsewhere) is

$$P_{int}(v) \triangleq P_{nd}(v) + (1 - P_{sbnd}(v))(1 - P_{nd}(v)), \quad (18)$$

and for the direct channel from the original source, we denote $P_{isd} \triangleq P_{sd} + (1 - P_{sbsd})(1 - P_{sd})$. Thus, the probability of successful retransmission becomes

$$P_{suc} = \left[(1 - \tau_s P_{isd}) \int_{v\in V} P_{sn}(v)\tau(v)P_{nd}(v)\rho(v)dv + \tau_s P_{sd}\right] \cdot \exp\left\{-\int_{v\in V} P_{sn}(v)\tau(v)P_{int}(v)\rho(v)dv\right\}. \quad (19)$$

Obviously, expression (19) reduces back to (6) or (7) in the limit when $P_{sbnd}(v)$ and P_{sbsd} are set to 1 and 0, respectively.

From here, an essentially similar derivation to that of the previous subsection can be repeated. In particular, once again, due to the linearity of (19) in τ_s, the optimum must be achieved at either $\tau_s = 0$ or $\tau_s = 1$. For the case of $\tau_s = 0$, considering the partial derivative with respect to the retransmission probability at a given location v_0 yields

$$\lim_{\Delta v\to 0} \frac{1}{\Delta v}\frac{\partial P_{suc}}{\partial \tau(v_0)} = P_{sn}(v_0)\rho(v_0) \cdot \left[P_{nd}(v_0) - P_{int}(v_0)\int_{v\in V} P_{sn}(v)\tau(v)P_{nd}(v)\rho(v)dv\right] \cdot \exp\left\{-\int_{v\in V} P_{sn}(v)\tau(v)P_{int}(v)\rho(v)dv\right\}. \quad (20)$$

Therefore, following the same considerations as in the lead-up to Theorem 1, we find that the optimal $\tau(v)$ that maximizes (19) is given by

$$\begin{cases} \tau(v) = 1 & \frac{P_{nd}(v)}{P_{int}(v)} > T_0 \text{ or } \frac{P_{nd}(v)}{P_{int}(v)} = T_0 \text{ and } v \in R_T \\ \tau(v) = 0 & \frac{P_{nd}(v)}{P_{int}(v)} < T_0 \text{ or } \frac{P_{nd}(v)}{P_{int}(v)} = T_0 \text{ and } v \notin R_T \end{cases} \quad (21)$$

where the threshold T_0 and region $R_T \subseteq \left\{v \,\middle|\, \frac{P_{nd}(v)}{P_{int}(v)} = T_0\right\}$ are now the solution of the integral equation

$$T_0 = \int_{\left\{v \middle| \frac{P_{nd}(v)}{P_{int}(v)} > T_0\right\} \cup R_T} P_{sn}(v)P_{nd}(v)\rho(v)dv. \quad (22)$$

For the case of $\tau_s = 1$, the calculation of the partial derivative of (19) with respect to $\tau(v_0)$ yields

$$\lim_{\Delta v\to 0} \frac{1}{\Delta v}\frac{\partial P_{suc}}{\partial \tau(v_0)} = P_{sn}(v_0)\rho(v_0) \cdot \left\{\left[P_{nd}(v_0) - P_{int}(v_0)\int_{v\in V} P_{sn}(v)\tau(v)P_{nd}(v)\rho(v)dv\right] \cdot (1 - P_{isd}) - P_{int}(v_0) \cdot P_{sd}\right\} \cdot \exp\left\{-\int_{v\in V} P_{sn}(v)\tau(v)P_{int(v)}\rho(v)dv\right\}, \quad (23)$$

which leads to the optimal $\tau(v)$ being defined by

$$\begin{cases} \tau(v) = 1 & \frac{P_{nd}(v)}{P_{int}(v)} > T_1 \text{ or } \frac{P_{nd}(v)}{P_{int}(v)} = T_1 \text{ and } v \in R_T \\ \tau(v) = 0 & \frac{P_{nd}(v)}{P_{int}(v)} < T_1 \text{ or } \frac{P_{nd}(v)}{P_{int}(v)} = T_1 \text{ and } v \notin R_T \end{cases} \quad (24)$$

with the threshold T_1 and region $R_T \subseteq \left\{v \,\middle|\, \frac{P_{nd}(v)}{P_{int}(v)} = T_1\right\}$ this time given by the solution of the integral equation

$$T_1 = \frac{P_{sd}}{1 - P_{isd}} + \int_{\left\{v \middle| \frac{P_{nd}(v)}{P_{int}(v)} > T_1\right\} \cup R_T} P_{sn}(v)P_{nd}(v)\rho(v)dv. \quad (25)$$

Due to space limits, we omit the detailed derivation of (21)–(22) and (24)–(25) and the rigorous existence proof of the respective integral equation solutions, which are obtained as straightforward extensions of the proof of Theorem 1.

We conclude this section by observing (without proof) that, while a retransmission strategy with either $\tau_s = 0$ or $\tau_s = 1$ can be better in any particular network instance (depending on the parameter values), it holds in general that, with everything else being equal, the optimal τ_s is monotonic in P_{sd}, i.e. there exists a threshold value of P_{sd}^{th} such that the optimal retransmission strategy has $\tau_s = 1$ if and only if $P_{sd} \geq P_{sd}^{th}$.

IV. Numerical Study

In this section, we demonstrate the performance of the uncoordinated strategy and its dependence on various network parameters numerically. First, we describe some assumptions that will be used in all the following scenarios.

We assume a signal propagation model that combines a path loss with a log-normal shadowing component, as follows:

$$\log P_r = \log P_t - L - \alpha \log d + \psi, \quad (26)$$

where P_r is the received power, P_t is the transmitted power, L is the power loss at a unit distance from the transmitter, d is the distance between the channel endpoints, α is the path loss exponent, and ψ is the random shadowing (Gaussian-distributed with a zero mean). Henceforth, we assume $\alpha = 2.7$ and $\sigma_\psi = 11.8dB$; these values are chosen to represent a typical city environment, based on the measurement study in [14]. We define a channel to be "good" if P_r is greater than some fixed threshold $P_{\min}$ necessary for decoding (e.g. due to receiver sensitivity and/or ambient noise). The probability of a channel to be good is thus $\Pr\{\psi \geq \log P_{\min} - \log P_t + L + \alpha \log d\}$, i.e. governed by the Gaussian distribution of ψ. Hence, there

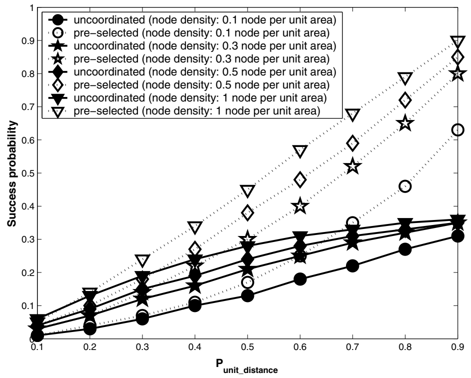

Fig. 1. Success probability vs. $P_{unit\_distance}$.

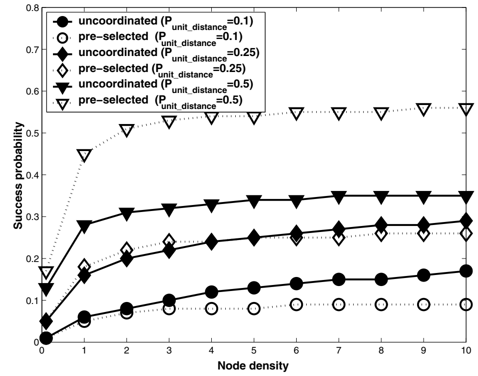

Fig. 2. Success probability vs. normalized node density.

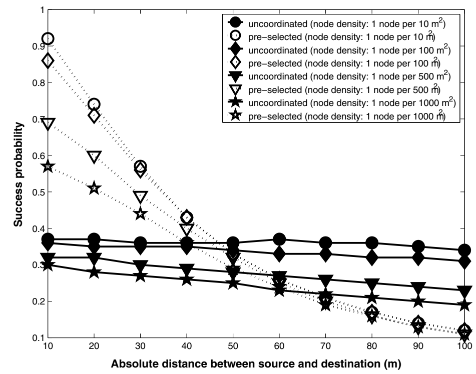

Fig. 3. Success probability vs. source-destination distance for fixed transmission power (set to have channel probability 0.5 at distance 30m).

is a one-to-one correspondence between P_t and the probability of a good channel over a unit distance, which we henceforth denote by $P_{unit\_distance}$; e.g., this probability is 0.5 for $\log P_t = \log P_{\min} + L$. We assume below that P_t and $P_{\min}$ are identical for all nodes in the network, and that the nodes are distributed on a two-dimentional plane with constant density $\rho(v) = \rho$ (i.e. a point Poisson process).

We choose to conduct all the following evaluations for case II, and to assume $P_{sd} = 0$, i.e. the direct channel has negligible probability to recover in the time slot immediately after a failed transmission. The reason for these choices is to focus on the most "hostile" possible scenario for our proposed method. Indeed, any combination of transmissions that leads to successful reception in case II does so in case I as well, but not vice versa; thus, the optimal success probability achievable in case II is always lower than for case I (or any mix thereof). Similarly, the success probabilities achievable with $P_{sd} = 0$ (which immediately implies $\tau_s = 0$) are a subset of those that are possible with $P_{sd} \geq 0$. Therefore, all the subsequent results represent a lower bound on the success probability achievable with an uncoordinated retransmission strategy. Incidentally, we point out that, since the propagation model implies that $P_{nd}(v)$ depends only on the distance to the destination, it follows that the optimal cooperation threshold (see Theorem 1) always translates to a circular region around the destination.

The results are presented in figures 1-3. Figure 1 shows the success probability of the optimal uncoordinated retransmission strategy as a function of $P_{unit\_distance}$ (equivalently, transmission power), for several values of density ρ, while Figure 2 plots the success probability versus ρ, for several values of $P_{unit\_distance}$. The values in both figures are normalized to a unit distance defined as the distance between the source and destination. Finally, in Figure 3, the success probability is shown as a function of the physical distance between source and destination, with a fixed transmission power and for several values of physical node density. For comparison purposes, we also calculate the expected success probability achievable by an optimally-located single cooperative relay selected *a priori*, which is simply the maximum $P_{sn}P_{nd}$ among all nodes in the network (obtained as an average of the maximum $P_{sn}P_{nd}$ value in a large sample of random networks of density ρ); this is labeled as "pre-selected" in the figures. Note that our comparison does not consider multiple-relay cooperation, which requires either multiple separate subchannels for the relayed signals or a significant *per-frame* overhead for choosing an optimal relay (as opposed to a single relay chosen *a priori*); as explained above, neither would be realistic in a highly dynamic ad hoc network.

We observe from figures 1-2 that, if $P_{unit\_distance}$ is large, then uncoordinated retransmission performs much worse than the coordinated single relay. This is expected, since a single well-placed relay will have very good channels to the source and destination, while the performance of an uncoordinated strategy is limited due to collisions. Indeed, the uncoordinated success probability is bounded by $\max_\lambda \lambda e^{-\lambda} = \frac{1}{e} \approx 0.368$ for case I (see (8)), and hence for case II as well. Incidentally, we note that this is the same bound as for the slotted ALOHA throughput [15], which features a similar uncoordinated access to a shared resource with collisions.

On the other hand, in the region of low $P_{unit\_distance}$, the uncoordinated strategy performs almost as well, or, for higher densities, even better than a coordinated single relay; this effect is best observed in the curves of $P_{unit\_distance} = 0.25$ in figure 2. We explain this effect as follows. For a single relay, regardless of the node density, the success probability cannot exceed that of an ideally placed relay mid-way between the source and destination, which is the square of the channel probability at half a unit distance. On the other hand, the uncoordinated strategy exploits high node densities by defining a small cooperation region around the destination (Figure 4).

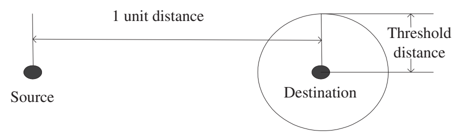

Fig. 4. The uncoordinated cooperation region.

Even if the channel quality at unit distance is low, for a sufficiently high density, the probability of overhearing by a node in the cooperation region will be reasonably high, due to the sheer number of nodes in the region (and the assumption of independent channels to different nodes); that node will then successfully retransmit it to the destination over a short distance. In fact, it can be shown that, for any fixed $P_{unit\_distance}$ (no matter how small), the success probability of the uncoordinated strategy will tend to $\frac{1}{e}$ as $\rho \to \infty$.

The same effect is even more pronounced in Figure 3, which is based on practical values of node density and transmission power. Here, each curve corresponds to an average density, ranging from 1 $\frac{\text{node}}{10m^2}$ (a dense network, e.g. a small room of people with wireless devices) to 1 $\frac{\text{node}}{1000m^2}$ (a sparse network in an open field or large hall). We set the transmission power to correspond to a channel probability of 0.5 at a distance of 30m, and vary the distance d_{sd} between the communicating endpoints (source and destination) between 10m and 100m. Clearly, as d_{sd} grows, the channel quality deteriorates and the performance of both the coordinated single relay and our uncoordinated retransmission strategy degrades. However, we clearly observe that the uncoordinated success probability degrades far more gracefully, and considerably outperforms coordinated relaying when the distance is large. The reason is that, with increasing d_{sd}, the normalized node density per unit distance (alternatively, the expected number of nodes in the general area between the source and destination) increases as well, as the square of d_{sd}, which does not matter to the single relay but crucially compensates for the reduction in channel quality with an uncoordinated retransmission strategy.

V. Conclusion

We have studied uncoordinated cooperative retransmission in ad-hoc wireless networks, where a node may retransmit an overheard frame without any prior agreement with its neighbors, thereby risking collision among multiple such retransmissions but eliminating the cooperation overhead if the transmission turns out successful. We modeled the respective tradeoff as a functional optimization problem, seeking the retransmission probability as a function of location to maximize the probability of successful reception, and provided a generic solution of the optimal cooperation region depending only on the *a priori* node density and the wireless signal propagation model. Our numerical evaluation demonstrated that the strategy performs especially well in scenarios with low channel quality (i.e. low transmission power or high level of noise) and high node density, in which case it even outperforms traditional (coordinated) relaying methods.

Our aim has been to present the potential benefits of uncoordinated cooperation via a simple strategy of a single immediate retransmission. Our analysis is completely generic and can be applied with any wireless signal propagation model. Nevertheless, further work is required to extend our results and alleviate some of our simplifying assumptions. First, we considered the optimization problem for a single frame in isolation, and the analysis must be extended to the scenario where multiple original frames can be transmitted in the network at the same time. In addition, more sophisticated strategies may be able to achieve an even higher performance, e.g. via carrier sensing for a small random period of time ahead of the cooperative retransmission, or by allowing several successive uncoordinated retransmission attempts before declaring a failure and reverting to the link-layer protocol. The extension of our analysis to more sophisticated strategies is left for future work.

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References

[1] A. Nosratinia, T.E. Hunter, and A. Hedayat. Cooperative communication in wireless networks. *IEEE Communications Magazine*, 42(10):74–80, October 2004.
[2] L. Lai, K. Liu, and H. El Gamal. The three-node wireless network: Achievable rates and cooperation strategies. *IEEE Transactions on Information Theory*, 52(3):805–828, March 2006.
[3] T.E. Hunter and A. Nosratinia. Diversity through coded cooperation. *IEEE Transactions on Wireless Communications*, 5(2):283–289, February 2006.
[4] A. Sendonaris, E. Erkip, and B. Aazhang. User cooperation diversity (parts I,II). *IEEE Transactions on Communications*, 51(11):1927–1948, November 2003.
[5] J.N. Laneman, D.N.C. Tse, and G.W. Wornell. Cooperative diversity in wireless networks: Efficient protocols and outage behavior. *IEEE Transactions on Information Theory*, 50(12):3062–3080, December 2004.
[6] J.N. Laneman and G.W. Wornell. Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks. *IEEE Transactions on Information Theory*, 49(10):2415–2425, October 2003.
[7] M. Janani, A. Hedayat, T.E. Hunter, and A. Nosratinia. Coded cooperation in wireless communications: Space-time transmission and iterative decoding. *IEEE Transactions on Signal Processing*, 52(2):362–371, February 2004.
[8] A. Murugan, K. Azarian, and H. El Gamal. Cooperative lattice coding and decoding in half-duplex channels. *IEEE Journal on Selected Areas in Communications*, 25(2):268–279, February 2007.
[9] P. Larsson. Selection diversity forwarding in a multihop packet radio network with fading channel and capture. *ACM Mobile Computing and Communications Review*, 5(4):47–54, October 2001.
[10] M. Zorzi and R.R. Rao. Geographic random forwarding (GeRaF) for ad hoc and sensor networks: Energy and latency performance. *IEEE Transactions on Mobile Computing*, 2(4):349–365, October 2003.
[11] S. Biswas and R. Morris. ExOR: Opportunistic multi-hop routing for wireless networks. In *Proc. ACM SIGCOMM*, Philadelphia, PA, August 2005.
[12] P. Nain, D. Towsley, B. Liu, and Z. Liu. Properties of random direction models. In *Proc. IEEE Infocom*, Miami, FL, March 2005.
[13] J.F.C. Kingman. *Poisson Processes*. Clarendon Press, 1992.
[14] S.Y. Seidel, T.S. Rappaport, S. Jain, M. Lord, and R. Singh. Path loss, scattering, and multipath delay statistics in four European cities for digital cellular and microcellular radiotelephone. *IEEE Transactions on Vehicular Technology*, 40(4):721–730, November 1991.
[15] D. Bertsekas and R. Gallager. *Data Networks*. Prentice-Hall, Englewood Cliffs, NJ, 2nd edition, 1992.