Connectivity of Wireless CSMA Multi-hop Networks

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***Abstract*—In this paper we consider the impact of interference on the connectivity of CSMA networks. First, it is shown that the aggregate interference experienced by any receiver in a CSMA network with arbitrarily distributed nodes is upper bounded. Then, we derive an equivalent transmission range for CSMA networks where any pair of nodes whose Euclidean distance is smaller than or equal to the transmission range are directly connected. Finally we give a sufficient condition on the transmission power required for a CSMA network with a total of n nodes *i.i.d.* on a $\sqrt{n} \times \sqrt{n}$ square following a uniform distribution to be asymptotically almost surely connected as $n \to \infty$ under the SINR model. It is shown that the transmission power only needs to be increased by a constant factor to combat interference and maintain connectivity compared with that considering a unit disk model without interference. This result is also in sharp contrast with previous results considering the connectivity of ALOHA networks under the SINR model.**

I. INTRODUCTION

Connectivity is one of the most fundamental properties of wireless multi-hop networks [1], [2], [3], and is a prerequisite for providing many network functions (e.g. routing, scheduling and localization). A network is said to be *connected* if and only if (iff) there is a (multi-hop) path between any pair of nodes. In a wireless network with many concurrent transmissions, signals transmitted at the same time may mutually interfere with each other. The SINR (signal-to-noise-plus-interference-ratio) model has been widely used to capture the impact of interference on network connectivity [4], [5].

Under the SINR model, the existence of a directional link between a pair of nodes is determined by the strength of the received signal from the transmitter, the interference caused by other concurrent transmissions and the background noise. Assume that all nodes transmit with the same power P and let $\boldsymbol{x}_k$, $k \in \Gamma$ be the location of node k, where Γ represents the set of indices of all nodes in the network. A node j can successfully receive the transmitted signal from a node i (i.e. node j *is directly connected to* node i) if the SINR at $\boldsymbol{x}_j$, denoted by $\text{SINR}\left(\boldsymbol{x}_i \to \boldsymbol{x}_j\right)$, is above a given threshold β, i.e.

$$\text{SINR}\left(\boldsymbol{x}_i \to \boldsymbol{x}_j\right) = \frac{P\ell\left(\boldsymbol{x}_i,\, \boldsymbol{x}_j\right)}{N_0 + \gamma \sum\limits_{k\in\mathcal{T}_i} P\ell\left(\boldsymbol{x}_k,\, \boldsymbol{x}_j\right)} \geq \beta \quad (1)$$

where $\mathcal{T}_i \subseteq \Gamma$ denotes the subset of nodes that are transmitting at the same time with node i and N_0 is the background noise power. The function $\ell\left(\boldsymbol{x}_i,\, \boldsymbol{x}_j\right)$ is the power attenuation from $\boldsymbol{x}_i$ to $\boldsymbol{x}_j$. The coefficient $0 \leq \gamma \leq 1$ is the inverse of the processing gain of the system and it weighs the impact of interference. In a broadband system using CDMA, γ depends on the orthogonality between codes used during concurrent transmissions and is smaller than 1; in a narrow-band system, γ equals to 1 [5], [6], [2]. Similarly, node i can receive from node j (i.e. node i is directly connected to node j) iff

$$\text{SINR}\left(\boldsymbol{x}_j \to \boldsymbol{x}_i\right) = \frac{P\ell\left(\boldsymbol{x}_j,\, \boldsymbol{x}_i\right)}{N_0 + \gamma \sum\limits_{k\in\mathcal{T}_j} P\ell\left(\boldsymbol{x}_k,\, \boldsymbol{x}_i\right)} \geq \beta \quad (2)$$

Therefore node i and node j are directly connected, i.e. a bidirectional link exists between node i and node j, iff both (1) and (2) are satisfied.

In [5] Dousse *et al.* used the above SINR model to analyze the impact of interference on connectivity based on the percolation theory. They considered a network where all nodes are Poissonly distributed on an infinite plane with a constant density λ and an attenuation function l with bounded support. By letting $\mathcal{T}_j = \Gamma / \{i, j\}$, i.e. all other nodes in the network are transmitting simultaneously with node i irrespective of their relative locations to $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$, it was shown that there exist γ' such that if $\gamma > \gamma'$ there is no infinite connected component in the network, i.e. the network does not *percolate*. However when $\gamma < \gamma'$, there exists $0 < \lambda' < \infty$ such that percolation can still occur when $\lambda > \lambda'$. An improved result by the same authors in [7] showed that under the more general conditions that $\lambda > \lambda_c$ and the attenuation function has unbounded support percolation occurs when $\gamma < \gamma'$, where λ_c is the critical node density above which the network with $\gamma = 0$ (i.e. *the unit disk model*) percolates [8, p48]. The above results suggest that percolation under the SINR model can happen iff γ is sufficiently small. In their network setting, it is assumed that each node transmits randomly and independently, irrespective of any nearby transmitter. This corresponds to the ALOHA-type multiple access scheme [2].

The ALOHA schemes have been superceded by more advanced multiple access schemes, e.g. CSMA and

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CSMA/CD (Carrier Sense Multiple Access With Collision Detection) [9], [10]. With CSMA schemes, each node checks the status of the wireless channel before sending a packet. If the channel is idle (i.e., no carrier is detected within its *carrier-sensing range*), then the node starts its transmission, otherwise the transmission is deferred, usually by a random amount of time, until the channel becomes idle [11]. Potential transmitters in the vicinity of an active transmitter are kept off. CSMA schemes can alleviate the interference by inhibiting concurrent nearby transmissions and are also shown to be superior to ALOHA schemes in performance.

In this paper, we analyze the connectivity of wireless CSMA multi-hop networks under the SINR model. Specifically consider a network with any number of nodes arbitrarily distributed in a finite area and a pair of nodes are directly connected iff both (1) and (2) are satisfied. Further the attenuation function assumes a power-law form, which is exactly the same model considered in [5], [7]. We show that when the carrier-sensing capabilities of wireless devices are considered, the interference experienced by any receiver in the network is upper bounded. Based on the above result, we further show that for an arbitrarily chosen SINR threshold, there exists a transmission range R_0 such that a pair of nodes are directly connected if their Euclidean distance is smaller than or equal to R_0. The above results, together with existing results on network connectivity for the unit disk model [1], [12], allow us to derive a sufficient condition for a CSMA network with a total of n nodes *i.i.d.* distributed on a $\sqrt{n} \times \sqrt{n}$ square following a uniform distribution to be asymptotically connected with probability one under the SINR model as $n \to \infty$.

The rest of this paper is organized as follows. Section II reviews related work. Section III describes system models. In Section IV, we first derive an upper bound on the interference in CSMA networks under the SINR model. Based on the upper bound, the transmission range R_0 is obtained. In Section V we give a sufficient condition for CSMA networks under the SINR model to be asymptotically connected with probability one as $n \to \infty$. Finally Section VI concludes this paper.

II. Related work

Extensive research has been done on connectivity problems using the well-know geometric graph and the *unit disk model*, which is usually obtained by randomly and uniformly distributing n nodes in a given area and connecting any two nodes iff their Euclidean distance is smaller than or equal to a certain threshold $r(n)$. This model corresponds to a special case of the SINR model in (1), i.e. when $\gamma = 0$ (perfect orthogonality, no mutual interference). In particular, it was shown that under the unit disk model and in a disk of unit area, the above network with $r(n) = \sqrt{\frac{\log n + c(n)}{\pi n}}$ is *a.a.s.* (asymptotically almost surely) connected as $n \to \infty$ iff $c(n) \to \infty$ [1], [12]. An event ξ_n, depending on n is said to occur *a.a.s.* if its probability tends to one as $n \to \infty$.

In [13], [3] the necessary condition for the above network to be a.a.s. connected was investigated under the more realistic *log-normal connection model*. Under the log-normal connection model, two nodes are directly connected if the received power at one node from the other node, whose attenuation follows the log-normal model, is greater than a given threshold. These results [13], [3] all rely on the assumption that the event that a node is isolated and the event that another node is isolated are independent.

The above studies however do not consider the impact of interference and the result obtained corresponds to a network with very light traffic load. Compared with the above studies, much less work has been done on network connectivity under the SINR model. In [14] Lebhar *et al.*, by enforcing each node to transmit in a particular time slot, showed how to emulate a unit disk graph in which each link operates under the SINR constraint. In [15] Avin *et al.* studied the connectivity problem from the perspective of scheduling. Specifically considering a two-dimensional grid network with a set of active links representing communication requests, assigning a color (time slots/frequencies) to each active link, they investigated how many colors are necessary in order for all active links to be simultaneously transmitting while satisfying the SINR constraint. The two papers [5], [7] discussed in Section I considered a network with nodes Poissonly distributed in $\Re^2$ under the SINR model and the ALOHA multiple access scheme and gave conditions under which the network percolates.

III. System Model

In this paper, two network models are considered. The results in Section IV are valid for a network with any number of nodes arbitrarily distributed in a finite area. In Section V we consider a network with a total of n nodes *i.i.d.* on a $\sqrt{n} \times \sqrt{n}$ square following a uniform distribution, i.e. the so-called *extended network model* [2]. It is assumed that all nodes in the network transmit at the same power P. Further, we consider that all transmitters are using the same channel, i.e. $\gamma = 1$, which corresponds to a narrow-band system [2], [5], [6].

A. Attenuation

We consider that the attenuation function ℓ in (1) only depends on Euclidean distance, namely $\ell : \Re^+ \to \Re^+$. Further we assume that $l(\cdot)$ is a power-law function [2], [6]:

$$l(s) = s^{-\alpha} \tag{3}$$

where s represents the Euclidean distance between a pair of nodes and α is the path-loss exponent, which typically varies from 2 to 6 [11, p139]. In this paper we assume $\alpha > 2$. The above assumptions on the attenuation function are widely used [2], [5], [6] and are also supported by measurement studies [11]. The results in this paper can be extended to the situation where $l(s) = \min\{1, s^{-\alpha}\}$ at the expense of some length but mostly straightforward discussions on the special case when $s^{-\alpha} < 1$. As commonly done in the connectivity analysis [1], [5], [7], [8], [12], the impact of small-scale fading is ignored.

B. Carrier sense

In CSMA networks, two nodes are allowed to transmit at the same time iff they can not detect each other's transmission, i.e. both $P\ell(\boldsymbol{x}_i, \boldsymbol{x}_j)$ and $P\ell(\boldsymbol{x}_j, \boldsymbol{x}_i)$ in (1) and (2) are under a certain detection threshold P_{th}. It then follows from (3) that the carrier-sensing range R_c, which determines the minimum Euclidean distance between two concurrent transmitters, is given by

$$R_c = (P/P_{th})^{1/\alpha} \tag{4}$$

IV. An Upper Bound on Interference in CSMA Networks and the Associated Equivalent Transmission Range

In this section, we first show that in CSMA networks and under the SINR model, the aggregate interference experienced by any receiver in the network is upper bounded. Based on the above result, we further show that for an arbitrarily chosen SINR threshold, there exists a transmission range R_0 such that a pair of nodes are directly connected if their Euclidean distance is smaller than or equal to R_0. The following theorem summarizes a major result of this paper:

Theorem 1. *Consider a CSMA network with nodes distributed arbitrarily on a finite area in $\Re^2$ where the carrier-sensing range is R_c, which is given by* (4)*, and each node transmits at the same power P. Assume that the attenuation function l is a power-law function given in* (3)*. Denote by r_0 the Euclidean distance between a receiver and its nearest transmitter in the network, which is also the intended transmitter for the receiver. Assuming $r_0 < R_c$, the maximum interference experienced by the receiver is smaller than or equal to $N(r_0) = N_1(r_0) + N_2$, where*

$$\begin{aligned} N_1(r_0) &= \frac{4P\left(\frac{\sqrt{3}}{2}\alpha R_c - r_0\right)\left(\sqrt{3}R_c - r_0\right)^{1-\alpha}}{3R_c^2(\alpha-1)(\alpha-2)} \\ &+ \frac{3P}{(R_c - r_0)^{\alpha}} + \frac{3P}{\left(\sqrt{3}R_c - r_0\right)^{\alpha}} + \frac{3P(R_c - r_0)^{1-\alpha}}{(\alpha-1)R_c} \end{aligned} \tag{5}$$

$$N_2 = \frac{3PR_c^{-\alpha}}{\alpha-1} + 3P\left(\sqrt{3}R_c\right)^{-\alpha} + \frac{3\alpha P\left(\frac{\sqrt{3}}{2}R_c\right)^{-\alpha}}{(\alpha-1)(\alpha-2)} \tag{6}$$

Proof: First note that a network on a finite area, denoted by $A \subset \Re^2$, can always be obtained from a network on an infinite area $\Re^2$ with the same node density and distribution by removing these nodes outside A. Such removal process will also remove all transmitters outside A. Therefore it follows that the interference experienced by a receiver in A is less than or equal to the interference experienced by its counterpart in a network in $\Re^2$. Thus it suffices to show that the interference experienced by a receiver in a network in $\Re^2$ is bounded.

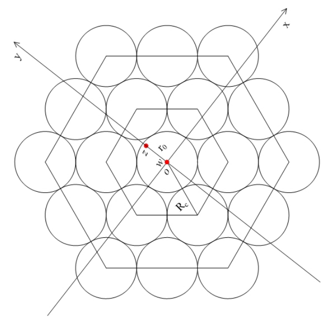

Figure 1. An illustration of the densest equal-circle packing.

In a CSMA network, any two concurrent transmitters are separated by at least an Euclidean distance R_c. Draw a circle of radius $R_c/2$ centered at each concurrent transmitter. Then the two circles centered at two closest transmitters cannot overlap except at a single point. Therefore the problem of determining the maximum interference can be transformed into one that determining the maximum number of non-overlapping equal circles that can be packed into $\Re^2$. In [16], it was shown that the densest equal circle packing is obtained by placing circle centers at the vertices of a hexagonal lattice [16, p8], as shown in Fig. 1.

Consider that an arbitrary receiver z is located at an Euclidean distance r_0 from its intended transmitter w. We construct a coordinate system such that the origin of the coordinate system is at w and z is on the $+y$ axis, as shown in Fig.1.

Partition the vertices of the hexagonal lattice into groups of increasing distances from the origin. The six vertices of the first group are within an Euclidean distance R_c to the origin. The $6m$ vertices in the m^{th} group are located at distances $((m-1)R_c, mR_c]$ from the origin.

Let I_1 be the total interference caused by transmitters, hereafter referred to as interferers, above the x-axis at node z. Using the triangle inequalities, it follows that $\|\boldsymbol{x}_i - \boldsymbol{z}\| \geq \|\boldsymbol{x}_i\| - r_0$ where $\boldsymbol{x}_i$ is the location of an interferer above the x-axis. Using some straightforward geometric analysis, it can be shown that among the $6m$ interferers in the m^{th} group, half of them are located above the x-axis. Among these interferers in the m^{th} group above the x-axis, three of them are at an Euclidean distance of exactly mR_c from the origin and the rest $3(m-1)$ interferers are at Euclidean distances between $[\frac{\sqrt{3}}{2}mR_c,\ mR_c]$. It then follows from (3) that

$$I_1 \leq \sum_{m=1}^{\infty}\left(\frac{3(m-1)P}{\left(\frac{\sqrt{3}}{2}mR_c - r_0\right)^{\alpha}} + \frac{3P}{(mR_c - r_0)^{\alpha}}\right) \tag{7}$$

In (7), it first can be shown that

$$\begin{aligned}
&\sum_{m=3}^{\infty} \frac{3(m-1)P}{\left(\frac{\sqrt{3}}{2}mR_c - r_0\right)^{\alpha}} \\
= \;& 3P\int_{2}^{\infty} \left(\lceil x\rceil - 1\right)\left(\frac{\sqrt{3}}{2}\lceil x\rceil R_c - r_0\right)^{-\alpha} dx \\
\leq \;& 3P\int_{2}^{\infty} (x-1)\left(\frac{\sqrt{3}}{2}xR_c - r_0\right)^{-\alpha} dx \\
= \;& \frac{4P\left(\sqrt{3}R_c - r_0\right)^{1-\alpha}\left(\frac{\sqrt{3}}{2}\alpha R_c - r_0\right)}{R_c^2(\alpha-1)(\alpha-2)} \qquad (8)
\end{aligned}$$

where $\lceil x\rceil$ denotes the smallest integer larger than or equal to x. Likewise, it can also be shown that

$$\sum_{m=2}^{\infty} \frac{3P}{(mR_c - r_0)^{\alpha}} \leq \frac{3P(R_c - r_0)^{1-\alpha}}{(\alpha-1)R_c} \qquad (9)$$

As a result of (7), (8), (9), it follows that

$$I_1 \leq N_1(r_0) \qquad (10)$$

Now we consider the total interference caused by interferers below the x-axis at node z, denoted by I_2. Let $\boldsymbol{x}_i$ be the location of an interferer below the x-axis. It follows from the triangle inequality $\|\boldsymbol{x}_i - \boldsymbol{z}\| \geq \|\boldsymbol{x}_i\|$ that

$$\begin{aligned}
I_2 \;\leq\; & \sum_{m=1}^{\infty}\left(\frac{3P}{(mR_c)^{\alpha}} + \frac{3(m-1)P}{\left(\frac{\sqrt{3}}{2}mR_c\right)^{\alpha}}\right) \\
= & \frac{3PR_c^{-\alpha}}{\alpha-1} + \frac{3P}{\left(\sqrt{3}R_c\right)^{\alpha}} + 3P\left(\frac{\sqrt{3}}{2}R_c\right)^{-\alpha}\sum_{m=3}^{\infty}\frac{(m-1)}{m^{\alpha}} \\
\leq & \frac{3PR_c^{-\alpha}}{\alpha-1} + 3P\left(\sqrt{3}R_c\right)^{-\alpha} + \frac{3\alpha P\left(\frac{\sqrt{3}}{2}R_c\right)^{-\alpha}}{(\alpha-1)(\alpha-2)} \qquad (11)
\end{aligned}$$

Combining (10) and (11), the theorem is proved. ∎

Remark 1. The assumption that $r_0 < R_c$ is valid in most wireless systems which not only require the SINR from a transmitter to a receiver to be above a certain threshold and also require the received signal from the transmitter to be of sufficiently good quality. However Theorem 1 does not critically depend on the assumption. In the situation that $r_0 \geq R_c$, so long as there exists a positive integer k such that $r_0 < kR_c$ the upper bound in Theorem 1 can be revised to accommodate the situation by changing the range of the summation in (8) and (9) from $[3,\infty]$ and $[2,\infty]$ to $[k+2,\infty]$ and $[k+1,\infty]$ respectively and revising (10) accordingly.

In the following discussion, we ignore the impact of background noise N_0 and use SIR for SINR for convenience. In cellular networks and dense sensor networks the background noise is typically negligibly small [2], [6]. The result can be easily extended to include the impact of background noise at the expense of more verbose discussions. The following result can be obtained from Theorem 1.

Corollary 1. *Under the same settings as Theorem 1, assume that the SIR threshold required for a successful transmission is β. There exists a transmission range $R_0 < R_c$ such that a pair of nodes are directly connected if their Euclidean distance is smaller than or equal to R_0. Further R_0 is given implicitly in the following equation*

$$PR_0^{-\alpha}/N(R_0) = \beta \qquad (12)$$

Proof: In Theorem 1, we have established that the interference experienced by a receiver z at an Euclidean distance r_0 from its transmitter w, denoted by $I(r_0)$ is upper bounded by $N(r_0)$. From (10) and (11), it can be shown that for $r_0 < R_c$ $N(r_0)$ is an increasing function of r_0. Note also that $Pr_0^{-\alpha}$ is a decreasing function of r_0. Therefore it can be shown that the SIR of a receiver at $r_0 \leq R_0$ from its transmitter, denoted by $SIR(r_0)$, satisfies

$$SIR(r_0) = \frac{Pr_0^{-\alpha}}{I(r_0)} \geq \frac{Pr_0^{-\alpha}}{N(r_0)} \geq \beta$$

i.e. the SIR at the receiver is greater than or equal to the threshold β.

By symmetry, when the packet transmission occurs in the opposite direction, i.e. from z to w, the interference generated by the set of nodes that are transmitting at the same time as z is also upper bounded by $N(r_0)$. Therefore the SIR at w is also greater than or equal to β.

Finally the existence of a (unique) solution to (12) can be proved by noting that $\frac{Pr_0^{-\alpha}}{N(r_0)} \to \infty$ as $r_0 \to 0$, $\frac{Pr_0^{-\alpha}}{N(r_0)} \to 0$ as $r_0 \to R_c^-$ and that $\frac{Pr_0^{-\alpha}}{N(r_0)}$ is a monotonically decreasing function of r_0. ∎

Corollary 1 relates the transmission range R_0 to the transmission power P. It allows the computation of R_0 given the transmission power P and the converse. A more convenient way to study the relation between P and R_0 is by noting that $P = P_{th}R_c^{\alpha}$ and considering R_0 as a function of R_c. Using (5) and (6) and letting $x = R_c/R_0$, (12) can be rewritten as

$$\begin{aligned}
\frac{1}{\beta} = \;& \frac{4\left(\sqrt{3}x-1\right)^{1-\alpha}\left(\frac{\sqrt{3}\alpha}{2}x-1\right)}{x^2(\alpha-1)(\alpha-2)} + \frac{3}{(x-1)^{\alpha}} \\
+ \;& \frac{3}{\left(\sqrt{3}x-1\right)^{\alpha}} + \frac{3(x-1)^{1-\alpha}}{(\alpha-1)x} + \frac{3}{\left(\sqrt{3}x\right)^{\alpha}} \\
+ \;& \frac{3}{x^{\alpha}(\alpha-1)} + \frac{3\alpha}{(\alpha-1)(\alpha-2)\left(\frac{\sqrt{3}}{2}x\right)^{\alpha}} \qquad (13)
\end{aligned}$$

V. Connectivity of CSMA Network

Based on the results derived in Section IV, in this section we consider the connectivity of a CSMA network with a total of n nodes uniformly *i.i.d.* on a $\sqrt{n} \times \sqrt{n}$ square under the SINR model. The main result of this section is summarized in the following theorem:

Theorem 2. *Consider a CSMA network with a total of n nodes* i.i.d. *on a $\sqrt{n} \times \sqrt{n}$ square following a uniform distribution. A pair of nodes are directly connected iff both* (1) *and* (2) *($\gamma = 1$ and $N_0 = 0$ in* (1) *and* (2) *) are satisfied. Further, the attenuation function $l : \Re^+ \to \Re^+$ in* (1) *and* (2) *is a power-law function given in* (3) *and the carrier-sensing condition is given in* (4). *Let $f(x)$ be a function of x and $f(x)$ is equal to the right hand side of* (13). *As $n \to \infty$, the above network is* a.a.s. *connected if the transmission power $P = P_{th} b^{\alpha} (\log n + c(n))^{\frac{\alpha}{2}}$, where $c(n) \to \infty$ as $n \to \infty$ and $\infty > b > 1$ is the solution to the following equation: $f(x) = 1/\beta$.*

Proof: We first show that there is a unique solution to (13) for any value of $\beta > 0$. It can be shown that $f(x)$ is a monotonically decreasing function of x for $x > 1$. Then noting that $f(x) \to \infty$ as $x \to 1$ and $f(x) \to 0$ as $x \to \infty$, it follows that for any value of $\beta > 0$, there exists a unique $\infty > b > 0$ such that $f(b) = \frac{1}{\beta}$.

Using the results in [1], [12] with proper scaling and coupling [8], it can be shown that for a network with a total of n nodes *i.i.d.* on a $\sqrt{n} \times \sqrt{n}$ square following a uniform distribution and a pair of nodes are directly connected iff their Euclidean distance is smaller than or equal to a given threshold $r(n)$ (i.e. the unit disk model), the network is a.a.s. connected as $n \to \infty$ iff $r(n) = \sqrt{\log n + c(n)}$ where $c(n) \to \infty$ as $n \to \infty$. Using the above result and letting $b = \frac{R_c}{R_0}$, Corollary 1 and Theorem 1, the result in the theorem follows. ■

The implication of Theorem 2 is that in CSMA networks, even when the impact of interference caused by multiple concurrent transmissions is considered, there exists a scheduling algorithm that allows as many as possible transmissions to be simultaneously active and in the meantime allows any pair of nodes in the network to be connected under an arbitrarily high SINR requirement, so long as the carrier-sensing capability is available. This result is in contrast to the ALOHA networks considered in [5], [7] in which percolation only occurs for a sufficiently small γ. Further, the transmission power only needs to be increased by a constant factor to combat interference and maintain connectivity compared with that in the unit disk model, in which no interference is considered.

VI. Conclusion

In this paper, we considered the impact of interference on connectivity of CSMA networks under the SINR model. We first showed that when the carrier-sensing capability is considered, the interference experienced by any receiver in a network is upper bounded. Based on this result, an equivalent transmission range is derived for CSMA networks with arbitrarily distributed nodes under the SINR model. Further, we consider a CSMA network with a total of n nodes uniformly and *i.i.d.* on a $\sqrt{n} \times \sqrt{n}$ square and gave a sufficient condition on the transmission power required for the network to be connected as $n \to \infty$. Compared with that in the unit disk model, the transmission power only needs to be increased by a constant factor to combat interference and maintain connectivity. This result is much more optimistic than those in [5], [7] considering the connectivity of ALOHA networks under the same SINR model.

We expect the sufficient condition derived in Theorem 2 to be reasonably tight because as $n \to \infty$, there is a very high probability that a node can be found within an Euclidean distance of $o(R_c)$ of the vertices of the hexagonal lattice in Fig. 1. It remains our future work to analytically investigate the tightness of the sufficient condition. Future work is also planned on the investigation of the necessary condition for the CSMA network considered in Theorem 2 to be connected. Both work can be possibly done by tighten the upper bound on interference in Theorem 1 by considering the impact of specific node distribution on the bound.

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