An Upper Bound on Transmission Capacity of Wireless CSMA Networks

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***Abstract*—Outage probability and transmission capacity are two metrics that are often used together to quantify the achievable capacity of decentralized wireless networks, where the outage probability measures the probability that a direct transmission fails and the transmission capacity measures the maximum spatial density of successful concurrent transmissions, subject to a constraint on the outage probability. In CSMA networks, spatial correlations between concurrent transmitters makes the analysis of the outage probability and the transmission capacity a challenging task. In this paper, we analyze the transmission capacity of CSMA networks subject to a designated outage probability constraint by first deriving an upper bound on the outage probability in CSMA networks subject to Rayleigh fading, which is applicable for any node distribution. On that basis, we provide a sufficient condition on the transmission power required to meet a designated outage probability constraint. Finally, we obtain an upper bound on the transmission capacity in CSMA networks satisfying a pre-determined outage probability constraint.**

I. Introduction

Studying the capacity of decentralized wireless networks (e.g. ad hoc networks) is an important problem and has attracted significant research interest in recent years [1]–[3].

A major approach in the area, pioneered by Gupta and Kumar [4], studies the so-called *transport capacity*, which quantifies the *end-to-end* throughput that can be transported over a physical distance for randomly chosen source-destination pairs in the network. Significant results have been obtained on characterizing the asymptotic scaling law of the transport capacity as the network becomes sufficiently large. It has been shown that by employing a nearest-neighbor routing scheme, the transport capacity that can be achieved is $\Theta\left(\sqrt{n}\right)$. The scaling law of the transport capacity provides a high-level insight on how different network features, e.g. network size, mobility of nodes and infrastructure support, affect the capacity. Notably, in [5], Grossglauser and Tse showed that mobility of nodes can be exploited to considerably increase the transport capacity to $\Theta\left(n\right)$ at the expense of delay, and [6], [7] showed that a transport capacity of $\Theta\left(n\right)$ can also be achieved by deploying (randomly placed) infrastructure/base stations.

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An alternative approach is to evaluate the so-called *transmission capacity* (TC). The transmission capacity, together with the *outage probability* (OP), have been used to quantify the achievable single-hop rates for decentralized wireless networks [1], [2], [8]–[10]. Specifically, the OP measures the probability that a single-hop (or direct) transmission fails, and the TC is defined as the maximum number of possible successful concurrent transmissions per unit area, subject to a constraint on the OP. The TC framework allows detailed study of the transmission capacity in terms of system parameters, e.g. fading and transmission success rate, which provides insights for a better design of lower layers of networks. This is generally very difficult to do by studying the transport capacity only.

Interference caused by concurrent transmissions is a major performance-limiting factor in a decentralized network. The randomness of interference is mainly attributable to 1) random node distribution and 2) medium access control (MAC) protocols. The combined effect of the above two factors determines the (often random) set of concurrent transmitters, hence interference. Analytical tools originating from the stochastic geometry theory have been developed in a number of papers [1], [2], [10] to characterize the performance of a decentralized network subject to interference. A considerably large body of work focuses on using Poisson point process (PPP) to model the spatial distribution of the concurrent transmitters, which is accurate when nodes are Poissonly distributed and ALOHA protocol is employed [2], [11]. For another widely used distributed MAC protocol - CSMA protocol, a constraint often exists on the minimum distance between concurrent transmitters due to the carrier-sensing requirements. This minimum distance constraint introduces a spatial correlation problem which implies that the location of a transmitter is no longer independent of the location of other concurrent transmitters. Therefore, even if all nodes are Poissonly distributed, the set of concurrent transmitters following the CSMA protocol no longer forms a PPP but a more complicate point process. Matérn hard-core point process is often used to model the spatial distribution of concurrent transmitters in CSMA networks. The analysis of such hard-core process is however quite challenging and existing work has only resulted in the analytical characterization of spatial averages of the main performance metrics, e.g. the average interference [12] and the average outage probability [10]. While the mean values of these parameters are important for capacity analysis, their distributions also have significant impact on the capacity which cannot be ignored.

In this paper we approach the problem of analyzing the transmission capacity and the outage probability by characterizing these performance impacting parameters using their bounds. This allows us to avoid the difficulty of accurately characterizing the exact distribution of these parameters, which are often analytically intractable. The contributions of this paper are:

1) We develop an upper bound on the OP in CSMA networks subject to Rayleigh fading, which is applicable for any node distribution.
2) Based on the above upper bound on the OP, we provide a sufficient condition on the transmission power required to meet a designated outage probability constraint.
3) We obtain an upper bound on the TC in CSMA networks satisfying a given outage probability constraint.

The paper is organized as follows. Section II reviews the related work. Section III describes the network model. In Section IV, we obtain an upper bound on the OP and the transmission power required for ensuring a designated OP. In Section V, we obtain an upper bound on the TC satisfying a prescribed OP constraint. Section VI concludes the paper.

II. Related work

In this section, we review work focusing on studying the TC of decentralized networks. Please refer to [9], [13], [14] and references therein for related work on studying the transport capacity.

The ALOHA-type of network has been thoroughly investigated and substantial progress has been made, see [2], [11] and references therein. In the absence of fading, the authors of [11] obtained a closed-form expression for the distribution of interference when the path-loss exponent, denoted by α, equals 4 and that distribution was shown to be the Lévy distribution. On that basis, Weber *et al.* [2] obtained the analytical expression for the TC. Also in [2], the authors gave lower and upper bounds on the OP for a generic value of α. In the presence of fading, the closed-form expression of the OP is given in [11] for Rayleigh fading, and approximations of the OP and the TC for more general fading are given in [2]. In [9], [11], interference and the OP are analyzed for ALOHA networks in which the locations of concurrent transmitters form a Poisson process. The property that the set of concurrent transmitters in ALOHA networks forms a Poisson process has greatly simplified the analysis. However the ALOHA schemes have become increasingly obsolete and have been superseded by more advanced MAC protocols, e.g. CSMA and CSMA/CD (Carrier Sense Multiple Access With Collision Detection) [15].

A major challenge in analyzing the performance of CSMA networks is that in CSMA networks, the locations of concurrent transmitters are no longer independent, i.e. two concurrent transmitters cannot be too close due to the carrier sensing requirements. Hence the set of concurrent transmitters can no longer be modeled by a Poisson process. In [16], the authors proposed to use the Matérn hard-core point process to model the set of concurrent transmitters in CSMA networks. While the hard-core-type point processes capture a key property of the concurrent transmitter set, i.e., two concurrent transmitters have to be separated by a minimum distance, such hard-core point process and the associated interference are very challenging to characterize analytically. Therefore, *approximation* is often used in order to obtain closed-form analytical results. In [3], [11], homogeneous PPP was used to approximate the spatial distribution of the set of concurrent transmitters in CSMA networks. In [12], the authors considered two types of hard-core point processes. They compared the *mean* interference generated by the two types of hard-core point processes with the mean interference generated by a PPP of the same node density. It was shown that the gap is negligible for one type, but increases exponentially with the minimum separation distance for the other hard-core point process. In [17], the distribution of concurrent transmitters is approximated by an inhomogeneous PPP whose local intensity depends on the distance from the desired transmitter. In [18], the authors provided approximations of the TC as the OP tends to zero.

In more recent work, Ganti *et al.* [8] analyzed asymptotic OP and TC for generic isotropic node distributions and generic fading as the spatial density of concurrent transmitters goes to zero. To be specific, they showed the procedure to obtaining two constants γ and κ such that, for general node distribution and fading distribution, the success probability p_s, viz. the complement of the OP, satisfies $p_s \sim 1 - \gamma\eta^{\kappa}$ when $\eta \to 0$, where η is the spatial density of concurrent transmitters ($f(x) \sim g(x)$ means that $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1$.).

In this paper, we solve the above difficulty involved in accurate modeling of the spatial distribution of concurrent transmitters in CSMA networks by finding an upper bound on the OP instead. Using the upper bound on the OP, the TC of the CSMA network satisfying the designated OP is obtained. Different from the above results [3], [8], [18], which have to resort to approximations of the spatial distribution of concurrent transmitters and empirical validation of the accuracy of such approximations, the results established in this paper are analytically rigorous.

III. Network model

We consider a network where nodes are distributed arbitrarily on an infinite plane [1]. Assume that all nodes transmit with a uniform power P. The value of P is chosen to meet the OP constraint and the choice of P is discussed later in Corollary 3. Let $\boldsymbol{x}_k$, $k \in \Gamma$, be the location of node k, where Γ represents the set of indices of all nodes in the network. Hereinafter, we also refer to a node by its location.

A. Radio propagation model

We consider a channel model consisting of a deterministic distance-dependent path-loss component and a random distance-independent component. Therefore, the interference

[1] As will be shown later, the density of nodes does not affect the validity of various bounds derived in this paper. Therefore we leave this parameter unspecified.

experienced at a receiver $\boldsymbol{x}_j$, when its intended receiver is $\boldsymbol{x}_i$, is given by

$$I(\boldsymbol{x}_i, \boldsymbol{x}_j) = \sum_{k \in \mathcal{T}_i} P h_{kj} \ell(\|\boldsymbol{x}_k - \boldsymbol{x}_j\|) \tag{1}$$

where $\mathcal{T}_i \subseteq \Gamma$ denotes the set of nodes transmitting at the same time as node $\boldsymbol{x}_i$, not including $\boldsymbol{x}_i$ itself. The path-loss assumes a power-law form, i.e. $\ell(\|\boldsymbol{x}_k - \boldsymbol{x}_j\|) = \|\boldsymbol{x}_k - \boldsymbol{x}_j\|^{-\alpha}$, where α is the path-loss exponent. We use the Rayleigh fading model for the random component, with which the received power $P h_{kj} \|\boldsymbol{x}_k - \boldsymbol{x}_j\|^{-\alpha}$ are exponentially distributed with mean $P \|\boldsymbol{x}_k - \boldsymbol{x}_j\|^{-\alpha}$. We further assume that random variables h_{kj} are *i.i.d.* This radio propagation model is widely used in literature [2], [10], [11], [18].

B. Outage probability and transmission capacity

The outage probability of a transmission from $\boldsymbol{x}_i$ to $\boldsymbol{x}_j$ is the probability that the signal-to-interference-plus-noise ratio (SINR) at $\boldsymbol{x}_j$ falls below the target SINR β, i.e.,

$$\text{OP} \triangleq \Pr\left[\frac{P h_{ij} \|\boldsymbol{x}_i - \boldsymbol{x}_j\|^{-\alpha}}{N + I(\boldsymbol{x}_i, \boldsymbol{x}_j)} < \beta\right] \tag{2}$$

where N is the background noise power and the success probability of a transmission is defined as the probability that the SINR at a receiver is greater or equal to the target SINR β, i.e. the complement of the OP. In dense sensor networks and cellular networks the background noise is typically negligibly small [8], [11], [13], therefore we ignore the background noise N in (2) in the following analysis. The TC is defined as the *maximum* spatial density of concurrent transmissions, subject to an outage probability constraint [2]. For a given constraint ϵ on the OP, the TC is given by

$$\text{TC}(\epsilon) \triangleq (1-\epsilon) \sup\{\eta : \text{OP} < \epsilon\} \tag{3}$$

where η is the density of concurrent transmitters in the CSMA network.

C. CSMA multiple-access protocol

The general idea of CSMA protocol is that nearby nodes will not be scheduled to transmit simultaneously. This exclusion rule is realized as follows: a node $\boldsymbol{x}_j$ is said to be in the contention domain of node $\boldsymbol{x}_i$ if the received power by $\boldsymbol{x}_i$ from $\boldsymbol{x}_j$ is above a certain detection threshold [10]. The node $\boldsymbol{x}_i$ is allowed to transmit if there is no other transmitting node in its contention domain, or in other words, the node $\boldsymbol{x}_i$ senses the medium idle. Considering the possible time-variation in the received power, in this paper, we consider that node $\boldsymbol{x}_j$ is *not* in the contention domain of node $\boldsymbol{x}_i$ if

$$\Pr\left[P h_{ij} \|\boldsymbol{x}_i - \boldsymbol{x}_j\|^{-\alpha} < P_{\text{th}}\right] \geq 1 - \sigma,\ \sigma \in (0,1) \tag{4}$$

where P_{th} is a detection threshold. One can regulate the probability that a node is allowed to transmit by tuning the value of σ. Since the received power is exponentially distributed, for a fixed value of σ, it follows from (4) that

$$1 - \exp\left(-\frac{P_{\text{th}}}{P \|\boldsymbol{x}_i - \boldsymbol{x}_j\|^{-\alpha}}\right) \geq 1 - \sigma \tag{5}$$

$$\|\boldsymbol{x}_i - \boldsymbol{x}_j\| \geq \left(\frac{P}{P_{\text{th}}} \times \ln\frac{1}{\sigma}\right)^{\frac{1}{\alpha}} \tag{6}$$

Inequality (6) specifies the minimum Euclidean distance between any two concurrent transmitters. We term this minimum distance as *the carrier-sensing range* in the presence of fading, denoted by R_c, i.e.,

$$R_c = \left(\frac{P}{P_{\text{th}}} \times \ln\frac{1}{\sigma}\right)^{\frac{1}{\alpha}} \tag{7}$$

IV. Analysis of the Outage Probability

In this section, we analyze the OP. Due to the difficulty involved in analyzing the exact spatial distribution of concurrent transmitters as mentioned in Section II, in this section, we focus on deriving an upper bound on the OP instead. The derived upper bound is applicable for any spatial node distribution. On that basis, we provide a sufficient condition on the transmission power to ensure that each link can meet a designated OP constraint ϵ.

Assume that each node transmits to its receiver which is located at a fixed Euclidean distance d_0 away. This assumption on transmitter-receiver arrangement is widely adopted and maybe easily relaxed without providing additional insight [2], [8]. Further, let $\mathcal{T}$ be the set of interferers, i.e. the set of concurrent transmitters other than the intended transmitter. Let d_i, $i \in \mathcal{T}$, be the Euclidean distance between the receiver and the i^{th} interferer. Let P_0 be the desired signal power and P_i, $i \in \mathcal{T}_i$, be the received power from the i^{th} interferer. With Rayleigh fading, the probability density function (pdf) of P_i is $f_{P_i}(x_i) = \frac{1}{\overline{P}_i} \exp\left(-\frac{x_i}{\overline{P}_i}\right)$, where $\overline{P}_i = P d_i^{-\alpha}$. The success probability of the link is then

$$\begin{aligned}
& 1 - \text{OP} \\
= \ & \Pr\left[\frac{P_0}{\sum_{i \in \mathcal{T}} P_i} \geq \beta\right] \\
= \ & \Pr\left[P_0 \geq \beta \sum_{i \in \mathcal{T}} P_i\right] \\
= \ & \int_0^\infty \cdots \int_0^\infty \exp\left(-\frac{\beta \sum_{i \in \mathcal{T}} x_i}{\overline{P}_0}\right) \times \prod_{i \in \mathcal{T}} f_{P_i}(x_i)\, dx_1 \cdots dx_{|\mathcal{T}|} \\
= \ & \frac{\overline{P}_0}{\beta \overline{P}_1 + \overline{P}_0} \times \int_0^\infty \cdots \int_0^\infty \exp\left(-\frac{\beta \sum_{i \in \mathcal{T}}^{i \neq 1} x_i}{\overline{P}_0}\right) \\
& \times \prod_{i \in \mathcal{T}}^{i \neq 1} f_{P_i}(x_i)\, dx_2 \cdots dx_{|\mathcal{T}|} \\
= \ & \prod_{i \in \mathcal{T}} \frac{\overline{P}_0}{\beta \overline{P}_i + \overline{P}_0} \\
= \ & \prod_{i \in \mathcal{T}} \frac{1}{\beta \left(\frac{d_i}{d_0}\right)^{-\alpha} + 1}
\end{aligned}$$

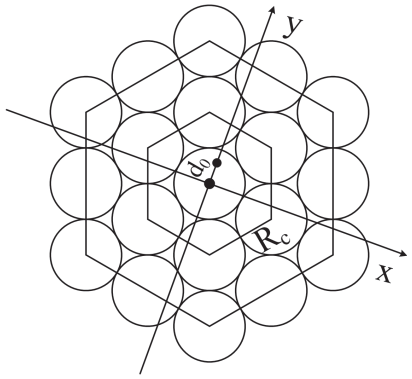

Figure 1. Densest circle packing

$$
\begin{aligned}
&= \exp\left(\ln\left(\prod_{i\in\mathcal{T}} \frac{1}{\beta\left(\frac{d_i}{d_0}\right)^{-\alpha}+1}\right)\right) \\
&= \exp\left(-\sum_{i\in\mathcal{T}} \ln\left(\beta\left(\frac{d_i}{d_0}\right)^{-\alpha}+1\right)\right) \\
&\geq \exp\left(-\sum_{i\in\mathcal{T}} \beta\left(\frac{d_i}{d_0}\right)^{-\alpha}\right) \qquad (8)
\end{aligned}
$$

where (8) follows from the fact that $\ln(x+1) \leq x$. Observe that RHS (right hand side) of (8) is an increasing function of d_is, which are the Euclidean distances between interferers and the desired receiver. A lower bound on RHS of (8) can be obtained when d_is are minimized subject to the constraint on the carrier-sensing, i.e. nearby transmitters have to be sufficiently separated in space.

In order to obtain a lower bound on RHS of (8), we construct a coordinate system such that the origin is at the desired transmitter and the receiver is on the $+y$ axis. Given the carrier-sensing range specified in (6) which prescribes the minimum distance between two concurrent transmitters, draw a circle of radius $R_c/2$ centered at each transmitter. Then the two circles centered at two closest transmitters cannot overlap except at a single point. It can then be shown that d_is in (8) are minimized when these circles are packed in $\Re^2$ in the densest way. Therefore the problem of finding a lower bound on RHS of (8) can be converted into a densest circle packing problem. The densest circle packing, i.e. fitting the maximum number of non-overlapping circles into $\Re^2$, is obtained by placing the circle centers at the vertices of a hexagonal lattice [19, p. 8], as shown in Fig. 1.

Group the vertices of the hexagonal lattice into tiers of increasing distances from the origin. The six vertices of the 1st tier are within a Euclidean distance R_c to the origin. The $6m$ vertices in the m^{th} tier are located at distances within $\left[\frac{\sqrt{3}}{2}mR_c, mR_c\right]$ from the origin.

Using the triangle inequalities, the Euclidean distance between the origin and an interferer above the x-axis is greater than or equal to $\frac{\sqrt{3}}{2}mR_c - d_0$, and the Euclidean distance between the origin and an interferer below the x-axis is greater than or equal to $\frac{\sqrt{3}}{2}mR_c$. It thus follows from (8) that

$$
\begin{aligned}
1-\text{OP} &\geq \exp\left(-\sum_{m=1}^{\infty} 3m\beta\left(\frac{\frac{\sqrt{3}}{2}mR_c}{d_0}\right)^{-\alpha}\right) \\
&\quad \times \exp\left(-\sum_{m=1}^{\infty} 3m\beta\left(\frac{\frac{\sqrt{3}}{2}mR_c-d_0}{d_0}\right)^{-\alpha}\right) \qquad (9)
\end{aligned}
$$

Consider the first factor in (9). Let U_m, $m=1,\ldots,\infty$ be random variables uniformly and *i.i.d.* in $[m-1/2, m+1/2]$. It follows from the convexity of $3m\beta\left(\frac{\frac{\sqrt{3}}{2}mR_c}{d_0}\right)^{-\alpha}$ as a function of m and Jensen's inequality that

$$
\begin{aligned}
&\sum_{m=1}^{\infty} 3m\beta\left(\frac{\frac{\sqrt{3}}{2}mR_c}{d_0}\right)^{-\alpha} \\
&= \sum_{m=1}^{\infty} 3\mathrm{E}\left[U_m\right]\beta\left(\frac{\frac{\sqrt{3}}{2}\mathrm{E}\left[U_m\right]R_c}{d_0}\right)^{-\alpha} \\
&\leq \sum_{m=1}^{\infty} \mathrm{E}\left[3U_m\beta\left(\frac{\frac{\sqrt{3}}{2}U_mR_c}{d_0}\right)^{-\alpha}\right] \\
&= \sum_{m=1}^{\infty} \int_{m-1/2}^{m+1/2} 3x\beta\left(\frac{\frac{\sqrt{3}}{2}xR_c}{d_0}\right)^{-\alpha} dx \\
&= \int_{1/2}^{\infty} 3x\beta\left(\frac{\frac{\sqrt{3}}{2}xR_c}{d_0}\right)^{-\alpha} dx \\
&= \frac{3\beta\left(\sqrt{3}/4\right)^{-\alpha}}{4(\alpha-2)}\left(\frac{R_c}{d_0}\right)^{-\alpha} \qquad (10)
\end{aligned}
$$

Using a similar method as above, we obtain an upper bound on the summation in the second factor of (9) as well, which is given by

$$
\begin{aligned}
&3\beta\left(\frac{\sqrt{3}R_c}{2d_0}-1\right)^{-\alpha} + 6\beta\left(\frac{\sqrt{3}R_c}{d_0}-1\right)^{-\alpha} \\
&+\frac{4\beta\left(\frac{5\sqrt{3}R_c}{4d_0}-1\right)^{1-\alpha}\left(\frac{5\sqrt{3}R_c}{4d_0}(\alpha-1)-1\right)}{(\alpha-1)(\alpha-2)\left(\frac{R_c}{d_0}\right)^2} \qquad (11)
\end{aligned}
$$

Treat the ratio $\frac{R_c}{d_0}$ as a variable and let

$$
\begin{aligned}
f\left(\frac{R_c}{d_0}\right) &\triangleq \frac{3\beta\left(\sqrt{3}/4\right)^{-\alpha}}{4(\alpha-2)}\left(\frac{R_c}{d_0}\right)^{-\alpha} \\
&+ 3\beta\left(\frac{\sqrt{3}R_c}{2d_0}-1\right)^{-\alpha} + 6\beta\left(\frac{\sqrt{3}R_c}{d_0}-1\right)^{-\alpha} \\
&+ \frac{4\beta\left(\frac{5\sqrt{3}R_c}{4d_0}-1\right)^{1-\alpha}\left(\frac{5\sqrt{3}R_c}{4d_0}(\alpha-1)-1\right)}{(\alpha-1)(\alpha-2)\left(\frac{R_c}{d_0}\right)^2}
\end{aligned}
$$

It is easy to observe that $f\left(\frac{R_c}{d_0}\right)$ is a decreasing function of $\frac{R_c}{d_0}$. Combining $f\left(\frac{R_c}{d_0}\right)$, (10), (11) with (9) yields an upper bound on the OP.

The above analysis is summarized in Theorem 1.

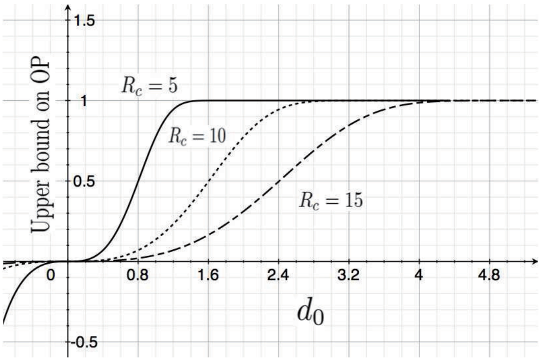

Figure 2. Upper bound on OP given by (12), for path-loss exponent $\alpha = 3$, SINR requirement $\beta = 10$.

Theorem 1. *In CSMA network with Rayleigh fading, the OP of all CSMA compliant transmissions is upper bounded, i.e.,*

$$OP \leq 1 - \exp\left(-f\left(\frac{R_c}{d_0}\right)\right) \tag{12}$$

Remark 2. Note that the upper bound on the OP given in Theorem 1 is independent of the node density. The implication of Theorem 1 is that in CSMA networks, a link is always feasible so long as its OP requirement is above $1 - \exp\left(-f\left(\frac{R_c}{d_0}\right)\right)$, no matter how dense the network is. Further, an arbitrarily low OP can be satisfied by carefully selecting the carrier-sensing threshold, represented by R_c, in (12). As shown in (7), the carrier-sensing range R_c can be tuned by tuning the transmission power P. This result is in sharp contrast with the ALOHA networks considered in [11, Eq. 3.29] and [2] where the success probability of a link decays exponentially as node density increases. Theorem 1 suggests however the success probability of CSMA networks is always above $\exp\left(-f\left(\frac{R_c}{d_0}\right)\right)$, irrespective of the node density.

Fig. 2 shows the OP upper bound given by (12), for different values of R_c and d_0.

Fig. 3 shows the outage probability versus the spatial density of all nodes. The simulation is conducted with nodes Poissonly distributed. We set the carrier-sensing range $R_c = 10$ and the network area to be a 200×200 square in the simulation. Fig. 3 shows that our bound is reasonably tight as the network becomes sufficiently dense. Compared with the results in [8], [18] which characterized the asymptotic behavior of the OP as the spatial density of concurrent transmitters goes to zero, we present another aspect of the OP as the transmitter density in CSMA networks becomes sufficiently high.

Based on Theorem 1, the following corollary can be obtained:

Corollary 3. *In a CSMA network, if the transmission power is set to satisfy* $P \geq \frac{P_{th}}{\ln\frac{1}{\sigma}}\left(d_0 b\left(\epsilon\right)\right)^{\alpha}$, *then any outage probability constraint* $0 < \epsilon < 1$ *can be met by all direct transmissions, i.e., OP* $< \epsilon$ *for all links, where* $b\left(\epsilon\right)$ *is the solution to the equation* $f\left(\frac{R_c}{d_0}\right) = -\ln\left(1-\epsilon\right)$.

Proof: Consider $f\left(\frac{R_c}{d_0}\right)$ to be a function of $\frac{R_c}{d_0}$. Firstly the existence of a unique solution to the equation $f\left(\frac{R_c}{d_0}\right) = -\ln\left(1-\epsilon\right)$ can be demonstrated by noting that $f\left(\frac{R_c}{d_0}\right) \to \infty$ as $\frac{R_c}{d_0} \to 0$, $f\left(\frac{R_c}{d_0}\right) \to 0$ as $\frac{R_c}{d_0} \to \infty$ and that $f\left(\frac{R_c}{d_0}\right)$ is a monotonically decreasing function of $\frac{R_c}{d_0}$. Denote the solution to $f\left(\frac{R_c}{d_0}\right) = -\ln\left(1-\epsilon\right)$ by $b\left(\epsilon\right)$ to emphasize the dependence on the outage probability constraint ϵ. To meet a given ϵ, according to (12), we need to have $f\left(\frac{R_c}{d_0}\right) \leq -\ln\left(1-\epsilon\right)$, or equivalently $\frac{R_c}{d_0} \geq b\left(\epsilon\right)$. Consider further that $R_c = \left(\frac{P}{P_{\text{th}}} \times \ln\frac{1}{\sigma}\right)^{\frac{1}{\alpha}}$ as given in (7), it is thus required that $P \geq \frac{P_{\text{th}}}{\ln\frac{1}{\sigma}}\left(d_0 b\left(\epsilon\right)\right)^{\alpha}$, which concludes the proof. ■

Corollary 3 provides a sufficient condition on the transmission power such that all CSMA compliant transmissions can meet a pre-determined outage constraint, which is a critical performance measure for decentralized wireless networks.

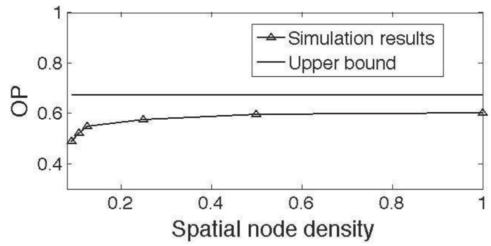

Figure 3. The outage probability versus the spatial node density, for path-loss exponent $\alpha = 3$, SINR requirement $\beta = 10$, transmission range $d_0 = 2$ and carrier-sensing range $R_c = 10$.

V. Transmission Capacity

In this section, using the upper bound on the OP given in Section IV, we derive an upper bound on the transmission capacity satisfying the designated OP constraint.

The approach we use is based on the observation that in interference-limited networks, each transmission "consumes" a certain area, i.e. there cannot be any concurrent transmitter in that area. Therefore the problem of finding an upper bound on the TC translates into a problem of finding a lower bound on the consumed area by each transmission under the outage probability constraint ϵ. From (7), the minimum Euclidean distance between any two concurrent transmitters is $R_c = \left(\frac{P}{P_{\text{th}}} \times \ln\frac{1}{\sigma}\right)^{\frac{1}{\alpha}}$. If the concurrent transmitters are packed in the densest way as described in Section IV (hence the TC is maximized), a pre-determined outage probability constraint ϵ can still be met by setting the transmission power to be the value suggested by Corollary 3. In this way, each transmission consumes a disk area with radius $\frac{R_c}{2}$ and with the transmitter located at the center of the disk. In addition, from the proof of Corollary 3 we have that to meet the OP constraint: OP $\leq \epsilon$, the inequality $\frac{R_c}{d_0} \geq b\left(\epsilon\right)$ needs to be satisfied. Therefore, the maximum spatial density of concurrent transmissions subject to the outage probability constraint ϵ is

$$\sup\left\{\eta : 1-\epsilon\right\} \leq \frac{1}{\pi\left(\frac{R_c}{2}\right)^2} \leq \frac{1}{\pi\left(\frac{1}{2}d_0 b\left(\epsilon\right)\right)^2} \tag{13}$$

Using the definition of TC given in (3), replacing $\sup\left\{\eta : 1-\epsilon\right\}$ by (13) yields an upper bound on the TC. We thus obtain the following Theorem, which is another main result of this paper.

Theorem 4. *In a CSMA network with Rayleigh fading, the TC satisfies*

$$TC \leq (1-\epsilon) \times \frac{1}{\pi\left(\frac{1}{2}d_0 b(\epsilon)\right)^2}$$

where $b(\epsilon)$ *is the solution to* $f\left(\frac{R_c}{d_0}\right) = -\ln(1-\epsilon)$ *for a pre-determined OP constraint* ϵ*.*

VI. Conclusion

In this paper we analyzed the outage probability and the transmission capacity of decentralized CSMA networks. Due to the intrinsic difficulties in analyzing the spatial distribution of concurrent transmitters in CSMA networks, we chose to derive an upper bound on the outage probability, instead of attempting to obtain the exact distribution. Based on that, we provided a sufficient condition on the transmission power required to meet a prescribed outage probability constraint. The results suggested that in CSMA networks, a link satisfying a designated outage probability constraint is always feasible by adjusting the transmission power, no matter how dense the network is. This result is in sharp contrast with previous results considering the ALOHA networks which showed that the success probability of a link decays exponentially as node density increases, i.e. at a sufficiently high node density, no link satisfying a designated outage probability constraint is feasible. Further, we obtained an upper bound on the transmission capacity of CSMA networks, subject to a pre-determined outage probability constraint.

References

[1] J. Andrews, S. Weber, M. Kountouris, and M. Haenggi, "Random access transport capacity," *IEEE Trans. Wireless Commun.*, vol. 9, no. 6, pp. 2101–2111, 2010.

[2] S. Weber, J. G. Andrews, and N. Jindal, "An overview of the transmission capacity of wireless networks," *IEEE Trans. Commun.*, vol. 58, no. 12, pp. 3593–3604, 2010.

[3] H. Q. Nguyen, F. Baccelli, and D. Kofman, "A stochastic geometry analysis of dense ieee 802.11 networks," in *Proceedings of IEEE INFOCOM*, pp. 1199–1207, 2007.

[4] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 388–404, 2000.

[5] M. Grossglauser and D. N. C. Tse, "Mobility increases the capacity of ad hoc wireless networks," *IEEE/ACM Trans. Netw.*, vol. 10, no. 4, pp. 477–486, 2002.

[6] P. Li and Y. Fang, "Impacts of topology and traffic pattern on capacity of hybrid wireless networks," *IEEE Trans. Inf. Theory*, vol. 8, no. 12, pp. 1585–1595, 2009.

[7] J.-w. Cho, S.-L. Kim, and S. Chong, "Capacity of interference-limited ad hoc networks with infrastructure support," *IEEE Commun. Letters*, vol. 10, no. 1, pp. 16–18, 2006.

[8] R. K. Ganti, J. G. Andrews, and M. Haenggi, "High-sir transmission capacity of wireless networks with general fading and node distribution," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 3100–3116, 2011.

[9] R. K. Ganti and M. Haenggi, "Interference and outage in clustered wireless networks," *IEEE Trans. Inf. Theory*, vol. 55, no. 9, pp. 4067–4086, 2009.

[10] F. Baccelli and B. Blaszczyszyn, *Stochastic Geometry and Wireless Networks Volume II : APPLICATION*. Paris: Now Publishers, 2009.

[11] M. Haenggi and R. K. Ganti, "Interference in large wireless networks," *Found. Trends Netw.*, vol. 3, no. 2, pp. 127–248, 2009.

[12] M. Haenggi, "Mean interference in hard-core wireless networks," *IEEE Commun. Letters*, vol. 15, no. 8, pp. 792–794, 2011.

[13] M. Haenggi, J. G. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, "Stochastic geometry and random graphs for the analysis and design of wireless networks," *IEEE J. Select. Areas Commun.*, vol. 27, no. 7, pp. 1029–1046, 2009.

[14] T. Yang, G. Mao, and W. Zhang, "Capacity of interference-limited three dimensional csma networks," in *Proceedings of IEEE ICC*, pp. 1–5.

[15] S. Kumar, V. S. Raghavan, and J. Deng, "Medium access control protocols for ad hoc wireless networks: A survey," *Ad Hoc Networks*, vol. 4, no. 3, pp. 326–358, 2006.

[16] A. Busson, G. Chelius, and Acm, "Point processes for interference modeling in csma/ca ad-hoc networks," in *Proceedings of ACM PE-Wasun*, pp. 33–40, 2009.

[17] G. Alfano, M. Garetto, and E. Leonardi, "New insights into the stochastic geometry analysis of dense csma networks," in *Proceedings of IEEE INFOCOM*, pp. 2642–2650, 2011.

[18] R. K. Ganti and J. G. Andrews, "A new method for computing the transmission capacity of non-poisson wireless networks," in *Proceedings of IEEE ISIT*, pp. 1693–1697, 2010.

[19] J. H. Conway and N. J. A. Sloane, *Sphere Packings, Lattices and Groups*. New York: Springer, 3rd ed., 1999.